Find the equation of ~~intersection~~ of the lines which pass through the intersection of lines $x+2y-19=0$ and $x-2y-3=0$. and are 5 unit from the point $(-2,4)$.

$$y-y_1=m(x-x_1)$$
$$y-2=m(x-x_1)$$

$$x+2y-19=0$$
$$x-2y-3=0$$
$$4y=16.$$
$$\boxed{y=4}$$

$$x+8-19=0.$$
$$\boxed{x=11}$$

Intersection $-$ $(11,4)$

$$5=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$y-4=m(x-11)$$
$$y-4=mx-11m$$
$$mx-y-11m+4=0.$$

Perpendicular distance from $(-2,4)$ is

$$\frac{-2m-\cancel{4}-11m+\cancel{4}}{\sqrt{m^2+1}}=\pm 5$$

$$m=\frac{\pm 5}{12}.$$

$$\frac{-13m}{\sqrt{m^2+1}} = \pm 5.$$

$$-13m = \pm 5\sqrt{m^2+1}$$

$$169m^2 = 25(m^2+1)$$
$$= 25m^2 + 25.$$

$$169m^2 - 25m^2 = 25.$$
$$144m^2 = 25.$$
$$m^2 = \frac{25}{144}.$$
$$m = \pm\frac{5}{12}$$

For $m = \frac{5}{12}$ eq^n of the line is.

$$y - 4 = \frac{5}{12}(x - 11)$$
$$12y - 48 = 5x - 55$$
$$5x - 12y - 7 = 0$$

For $m = -\frac{5}{12}$

$$y - 4 = -\frac{5}{12}(x - 11)$$
$$12y - 48 = -5x + 55.$$
$$5x + 12y - 48 - 55 = 0.$$
$$5x + 12y - 103 = 0.$$

Q. Find the distance between the parallel lines.. $2x + 3y - 25 = 0$, $4x + 6y + 11 = 0$.

$$x = 0 \quad y = \frac{25}{3}$$

$$\text{Point} = \left(0, \frac{25}{3}\right)$$

$$d = \frac{\left|4(0) + 6\left(\frac{25}{3}\right) + 11\right|}{\sqrt{16+36}}$$
$$= \frac{61}{\ldots}$$

6/8/15..

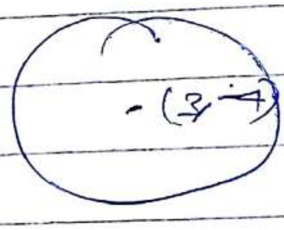
~~$3x-2y-1=0$.~~

Q. Find the eqn of the circle whose centre is = (3,-4) and passes through the intersection of the lines $3x-2y-1=0$. & $4x+y-27=0$.

$$3x-2y-1=0.$$
$$8x+2y-54=0.$$
$$11x-55=0$$
$$11x=55$$
$$x=5$$

Intersection

$$15-2y=1$$
$$2y=14$$
$$y=7$$



$$x^2+y^2+2(-3)x+2(4)y-c=0.$$
$$x^2+y^2-6x+8y-c=0.$$

$x^2+y^2-6x+8y-10$

$$25+49-30+56=c.$$
$$44+56=c$$
$$\boxed{c=100}$$

25
19
44

$$x^2+y^2-6x+8y-100=0$$

IInd method

$$cp = \text{radius} = \sqrt{(5-3)^2+(7+4)^2}$$
$$= \sqrt{4+121} = \sqrt{12}$$

$(x-h)^2+(y-k)^2=a^2$

$(x-3)^2+(y+4)^2=125.$

$x^2-6x+9+y^2+8y+16=125$

Q. Find the coordinates of the centre and radius of the circle.

$4(x^2+y^2)+12ax-6ay-a^2=0$

$4x^2+4y^2+12ax-6ay-a^2=0.$

$x^2+y^2+3ax-\frac{3}{2}ay-\frac{a^2}{4}=0.$

$2g=3a.$ $\qquad$ $-\frac{3}{2}a=2f.$

$\boxed{g=\frac{3}{2}a}$ $\qquad$ $\boxed{f=-\frac{3}{4}a.}$ $\qquad$ c=

Centre $=\left(-\frac{3}{2}a,\ \frac{3}{4}a\right).$

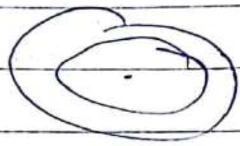
radius $=\sqrt{\frac{9}{4}a^2+\frac{9}{16}a^2+\frac{a^2}{4}}$

$=\sqrt{\frac{36a^2+9a^2+4a^2}{16}}$

$=\sqrt{\frac{49a^2}{16}}=\frac{7a}{4}$

$=$

Q. Find the equation of the circle concentric with the circle $2x^2+2y^2+8x+10y-39=0$ having its area equal to 16π.



$\cancel{\pi} r^2 = 16\cancel{\pi}$

$r = 4$

$2x^2+2y^2+8x+10y-39=0$

$x^2+y^2+4x+5y-\frac{39}{2}=0$

$2g=4$

$g=2 \quad c=\left(-2, -\frac{5}{2}\right)$

$2f=5$

$f=\frac{5}{2}$

$\left(x+2\right)^2+\left(y+\frac{5}{2}\right)^2=16$

$x^2+\frac{1}{4}+x+y^2+\frac{25}{4}+5y-16$

$x^2+y^2+x+5y+\frac{26}{4}-16=0$

$\cancel{x^2+y^2+x+5y+1}$

$4x^2+4y^2+4x+20y+26-4=0$

$4x^2+4y^2+4x+20y+23=0$

$x^2+4+4x+y^2+\frac{25}{4}+5y-16=0$

$x^2+y^2+4x+5y-12+\frac{25}{4}=0$

Q. Find the equation of the circle passing through the points (5, 7), (8, 1) and (1, 3). Find also its centre & radius.

Sol:- let the general eqⁿ of circle be

$$x^2+y^2+2gx+2fy+c=0$$

$$25+49+10g+14f+c=0$$
$$64+1+16g+2f+c=0$$
$$1+9+2g+6f+c=0$$

$$74+10g+14f+\cancel{c}=0$$
$$\underline{65+16g+2f+\cancel{c}=0}$$
$$9-6g+12f=0.$$

$$\cancel{6g+12f}\quad 6g-12f=9.$$

$$2g-4f=3$$

$$65+16g+2f+c=0$$
$$\underline{10+2g+6f+c=0}$$
$$55+14g-4f=0.$$

$$14g-4f=-55$$

$$2g-4f=3$$
$$\underline{14g-4f=-55}$$
$$-12g=58$$

$$g=\frac{58}{12}=\frac{29}{6}$$

$$\frac{58}{3}-9=4f.$$

$$-67=4f$$

$$\frac{29}{2}-4f=3.$$

$$4f=\frac{29-6}{2}$$

$$12f-6\left(\frac{-29}{6}\right)=$$

Q. Find the equation of the circle which passes through the points $(5,-8)$, $(-2,+9)$ and $(2,1)$. Find the centre and radius.

Let the general equation of the circle is $x^2+y^2+2gx+2fy+c=0$ — (1)

For $(5,-8)$ $25+64+10g-16f+c=0$ — (2)

$10g-16f+c=-89$

For $(-2,+9)$ ~~$4+81-18f+c=0$~~ — (3)

$4+81-4g+18f+c=0$

For $(2,1)$ $4+1+4g+2f+c=0$ — (4)

$-4g+18f+c=-85$ (3) $4g+2f+c=-5$ (4)

~~$4g-18f-c=85$~~

(3)+(4)

$\cancel{-4g+8f+c=-85}$

$\cancel{4g+2f+c=-5}$

$\cancel{10f+2c=}$

(3)-(4)

$-4g+18f+c=-85$

$\underline{\overset{-}{4g}\ \overset{-}{+}\ 2f\ \overset{-}{+}\ c=\overset{+}{-}5}$

$-8g+16f=-80$

(2)-(3)

$10g-16f+c=-89$

$\underline{\overset{+}{-4g}\ \overset{-}{+}\ 18f\ \overset{-}{+}\ c=\overset{+}{-}85}$

$14g-34f=-4$

$-8g + 16f = -80$
$14g - 34f = -4$

$g = 58$
$f = 24$
$c = -285$

$-42g + 96f = -480$
$42g - 102f = -12$

$-6f = -492$
$\boxed{f = 82}$??

Equation is
$x^2+y^2+116x+48y-285=0$
$c = (-58,-24)$

Q. Find the equation of the circle which pass through the points $(3,-2)$, $(-2,0)$ and has it on the line $2x-y=3$.

Equation of the circle is
$x^2+y^2+2gx+2fy+c=0$ — (1)

For $(3,-2)$ $9+4+6g-4f+c=0$
$6g-4f+c=-13$ — (2)

For $(-2,0)$ $4+0-4g+c=0$
$-4g+c=-4$ — (3)

centre $(-g,-f)$ lies on $2x-y=3$
$-2g+f=3$ — (4)

$$6g - 4f + c = -13$$
$$-4g \quad 0 + c = -4$$

$$10g - 4f = -9$$
$$-8g + 4f = 12$$

$$2g = 3$$
$$\boxed{g = 3/2}$$

$$-4 \times \frac{3}{2} + c = -4$$
$$-6 + c = -4$$
$$c = -4 + 6$$
$$\boxed{c = 2}$$

$$g = 3/2$$
$$f = 6$$
$$c = 2$$

$$10 \times \frac{3}{2} - 4f = -9$$
$$15 - 4f = -9$$
$$-4f = -9 - 15$$
$$-4f = -24$$
$$\boxed{f = 6}$$

Find the eq^n of the circle which pass through the points $(1,-2)$, $(4,-3)$ and has its centre on the line $3x + 4y - 7 = 0$

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$1 + 4 + 2g - 4f + c = 0$$
$$2g - 4f + c = -5 \quad \text{—— (i)}$$

For $(4, -3)$

$$16 + 9 + 8g - 6f + c = 0.$$

$$8g - 6f + c = -25 \quad \text{— (2)}$$

centre lies on $3x + 4y - 7 = 0$ — (3).

$$-3g - 4f = 7$$

$$2g - 4f + \cancel{c} = -5.$$

$$8g - 6f + \cancel{c} = -25$$

$$- \quad + \quad - \quad +$$

$$-6g + 2f = 20.$$

$$-\cancel{6g} - 8f = 14$$

$$-\cancel{6g} + 2f = 20$$

$$+ \quad - \quad -$$

$$+10f = +6.$$

$$\boxed{f = \frac{3}{5}}$$

$$-3g - 4\left(\frac{3}{5}\right) = 7.$$

$$-3g = 7 + \frac{12}{5}$$

$$\boxed{g = -\frac{47}{15}}$$

$$2\left(\frac{-47}{15}\right) - 4\left(\frac{3}{5}\right) + c = -5$$

$$\frac{-94}{15} - \frac{12}{5} + c = -5.$$

$$\frac{-94 - 36}{15} + c = -5$$

$$-\frac{\cancel{130}^{26}}{\cancel{15}_{3}} + c = -5$$

11/8/15.

centre $(-g, -f)$

Centre lies on x axis $y = 0 \Rightarrow -f = 0$.

$f = 0$.

" " y axis $x = 0 \Rightarrow -g = 0$

$\boxed{g = 0}$

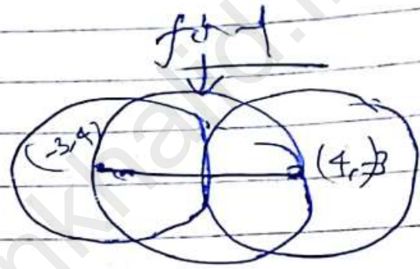
Q. Find the equation of the circle drawn on the line joining the centres of the circles as diameters $x^2+y^2-6x+8y-20=0$ and

$x^2+y^2+8x-6y=0$.

$x^2+y^2-6x+8y-20=0$.

$g=-3 \quad f=4$

Centre $(+3, -4)$



$x^2+y^2+8x-6y=0$.

$g=4 \quad f=-3$

Centre $(-4, +3)$

$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$

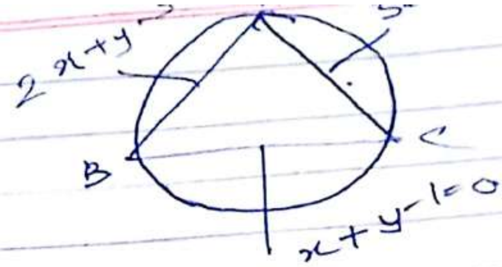
$(x-3)(x+4)+(y+4)(y-3)=0$.

$x^2+4x-3x-12+y^2-3y+4y-12=0$

$\boxed{x^2+y^2+x+y-24=0}$

Q. Find the equation of the circle circumscribing the triangle formed by the lines $2x+3y-5=0$, $x+y-1=0$ and $3x+2y-5=0$

$2x + y = 3$.
$x + y = 1$
$\boxed{x = 2}$ $\qquad x + y = 1$ $\qquad \boxed{y = -1}$



B $(2, -1)$

$2x + 2y = 2$ $\qquad x + y = 1$.
$3x + 2y = 5$ $\qquad \boxed{y = -2}$
$\boxed{x = 3}$ $\qquad$ C $(3, -2)$

$3x + 2y = 5$.
$4x + 2y = 6$ $\qquad y = 3 - 2$ $\qquad$ A$(1, 1)$
$-x = -1$ $\qquad \boxed{y = 1}$
$\boxed{x = 1}$

Centroid $\left(\frac{2+3+1}{3}, \frac{-1-2+1}{3}\right)$

$= \left(2, \frac{-2}{3}\right)$ $\qquad \left(g = -2 \quad f = \frac{2}{3}\right)$

$x^2 + y^2 - 4x + \frac{4}{3}y + c = 0$

$g = -$
$f = -$
$c = 1$

$4 + \frac{4}{9} - 8 - \frac{8}{3} + c = 0$.

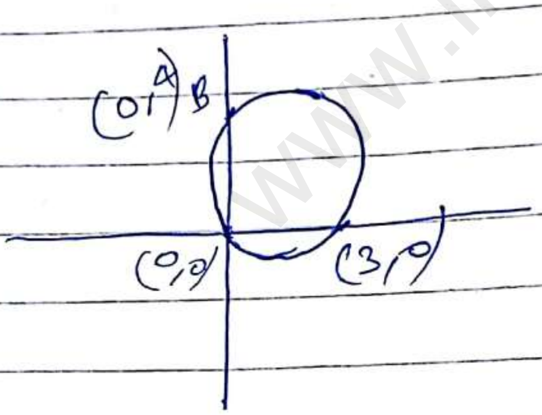
$-4 - \frac{20}{9} + c = 0$.

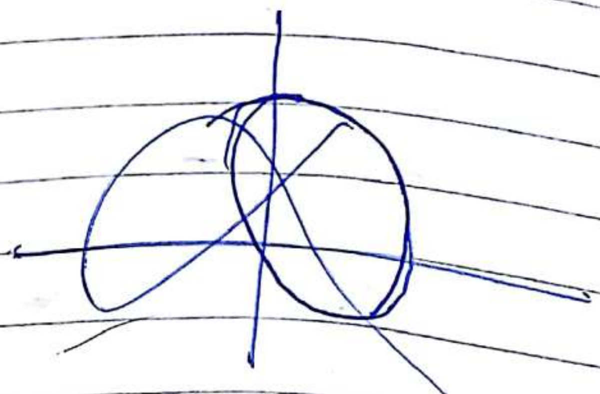
$1 + 1 + 2g + 2f + c = 0$

$\frac{-36 - 20}{9} + c = 0$. $\qquad 2g + 2f + c = -2$

$-\frac{56}{93}$

Q. Find the equation of circle passes through origin and cuts off intercepts from the axes equal to 3 and 4.





$c = 0$

$g = -\frac{3}{2}$

$f = -2$

$x^2 + y^2 - 3x - 4y = 0$

Let the eqn of circle be

$x^2+y^2+2gx+2fy+c=0$

One point is $(0,0)$ $\therefore$ $\boxed{c=0}$

$8f + c = -16$

$8f = -16$ $\quad (\because c = 0)$.

$\boxed{f = -2}$

For $(3,0)$

$9 + 0 + 6g + c = 0.$

$6g = -9.$

$\boxed{g = -\frac{3}{2}}$

therefore eqn of the circle is.

$x^2 + y^2 + 2\left(-\frac{3}{2}\right)x + 2(-2)y = 0$

$\boxed{x^2 + y^2 - 3x - 4y = 0}$

CONIC —

Conic — A conic is a locus of a p[oint] moves so that its distance from a fixed point is in a constant ratio to its distance from a fixed straight line.

Fixed point is called focus - S

Fixed st. line is called directrix - MZ

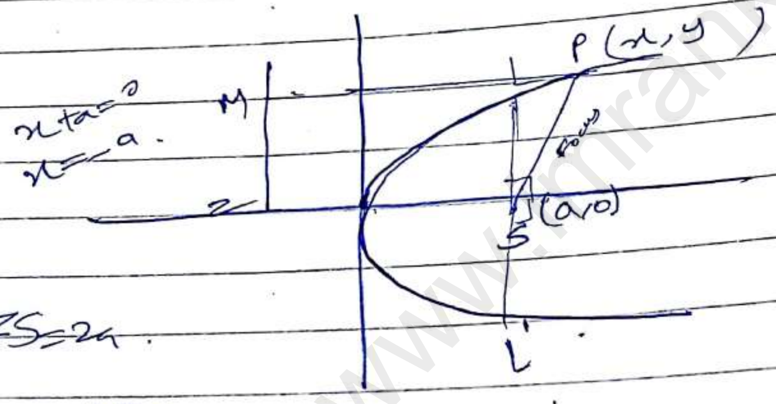
constant ratio is called eccentricity - e

A conic is a parabola if $e = 1$.

A conic " hyperbola $e > 1$

A conic " ellipse $e < 1$

Equation of a parabola $y^2 = 4ax$.



$$PS = \sqrt{(x-a)^2 + y^2}$$

$$PM = \frac{x+a}{\sqrt{(1)^2}}$$

$$= x + a$$

Let $ZS = 2a$.

$ZA = AS$.

$\boxed{AS = a}$

LL' is called latus rectum

Latus rectum =

For parabola $e=1$

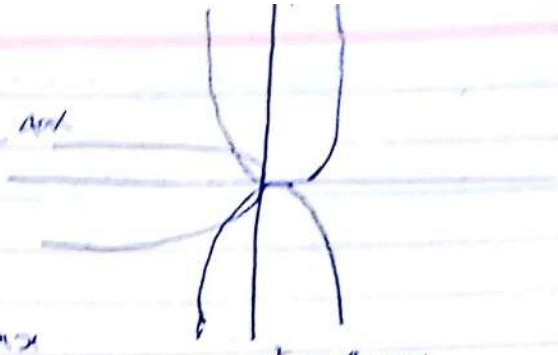
$PS = PM$

$\sqrt{(x-a)^2+y^2} = x+a$.

Squaring both sides,

$\cancel{x^2}+\cancel{a^2}-2ax+y^2 = \cancel{x^2}+\cancel{a^2}+2ax$.

$\boxed{y^2 = 4ax}$ for x-axis.



Equation	$y^2=4ax$	$y^2=-4ax$	$x^2=4ay$	$x^2=-4ay$
Vertex	$(0,0)$	$(0,0)$	$(0,0)$	$(0,0)$
Focus	$(a,0)$	$(-a,0)$	$(0,a)$	$(0,-a)$
Latus rectum	$4a$	$4a$	$4a$	$4a$
Axis	$y=0$	$y=0$	$x=0$	$x=0$
Directrix	$x+a=0$	$x-a=0$	$y+a=0$	$y-a=0$

Q. Find the equation of the parabola whose focus is $(3,0)$ and directrix is $3x+4y=1$

$PM = \dfrac{3x+4y-1}{\sqrt{9+16}}$

$PS = \sqrt{(x-3)^2+y^2}$

$PM = \dfrac{3x+4y-1}{5}$

$= \sqrt{x^2+9-6x+y^2}$

$x^2+y^2-6x+9 = \dfrac{9x^2+(4y-1)^2+6x(4y-1)}{25}$

$$x^2+y^2-6x+9=\frac{9x^2+16y^2+1+8y+24xy-6x}{25}$$

$$25x^2+25y^2-150x+225=9x^2+16y^2+24xy-6x+8y+1$$

$$\cancel{16x^2-15y^2-144x+224=0}$$

$$16x^2+9y^2-144x-24xy+224=0.$$

Q. Find the equation of the parabola whose focus is $(-3,2)$ and the directrix is $x+y=4$

$$PM=\frac{x+y-4}{\sqrt{1^2+1^2}}=\frac{x+y-4}{\sqrt{2}}$$

$$PS=\sqrt{(x+3)^2+(y-2)^2}.$$

$$PM=PS.$$

$$\frac{(x+y-4)^2}{2}=(x+3)^2-(y-2)^2$$

$$\frac{(x+y)^2+16-2(4)(x+y)}{2}=x^2+9+6x-(y^2+4-$$

$$x^2+y^2+2xy+16-8x-8y=2x^2+18+12x-2y^2-8+8x.$$

$$-x^2+3y^2+2xy-28x-6=0.$$

Q. Find the equation of the parabola whose focus is $(1,-1)$ and whose vertex is $(2,1)$

$$PS = \sqrt{(x-1)^2+(y+1)^2}.$$

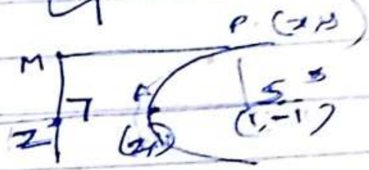
Slope of the axis

$(A-S) = \frac{1+1}{2-1} = 2.$

$\boxed{m_1 m_2 = -1}$

$2 m_2 = -1$

$\boxed{m_2 = \frac{-1}{2}}$



A is midpoint of Z & S

$2 = \frac{a+1}{2}, \quad 1 = \frac{b-1}{2}$

$(a=3, \quad b=3)$

$y - y_1 = m(x - x_1)$

$y - 3 = \frac{-1}{2}(x-3)$

$2y - 6 = -x + 3$

Eq^n of directrix

$\cancel{x+2y} = \quad x + 2y - 9 = 0.$

$PM = \frac{x+2y-9}{\sqrt{1+4}}$

$= \frac{x+2y-9}{\sqrt{5}}.$

$PS = PM \quad (\because e =$

$\frac{x+2y-9}{\sqrt{5}} = \sqrt{(x-1)^2+(y+}$

$\frac{(x+2y-9)^2}{5} = (x-1)^2 + (y+1)^2,$

$\frac{(x+2y)^2 + 81 - 2(9)(x+2y)}{5} = x^2+1-2x+y^2$

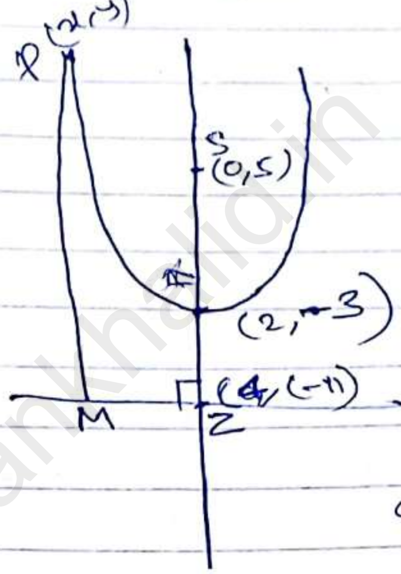
$x^2+4y^2+4xy+81-18x-36y$

$= 5x^2+5-10x+5y^2+5+1$

$$-4x^2 - y^2 + 4xy + 8x - 46y + 71 = 0$$

$$4x^2 + y^2 - 4xy + 8x + 46y - 71 = 0$$

Q. Find the eq^n of the parabola whose ~~faces~~ focus is $(0,5)$ and vertex is $(2,-3)$



$m_1 = \frac{-3-5}{2}$

$= \frac{-8}{2} = -4$.

$m_1 m_2 = -1$.

$-4 m_2 = -1$

$\boxed{m_2 = +\frac{1}{4}}$

$\frac{5+b}{2} = -3$

$5+b = -6$

$b = -11$

$\frac{0+a}{2} = 2$

$0+a = 4$

$(a=4)$

Using point slope form

$y - y_1 = m(x - x_1)$

$y + 11 = +\frac{1}{4}(x+4)$

$4y + 44 = +x + 4$

$4y - x = -48$.

~~$x + 4y + 48 = 0$~~.

$x - 4y - 48 = 0$.

$PM = \frac{x + 4y + 48}{\sqrt{1+16}}$

$= \frac{x + 4y + 48}{\sqrt{17}}$.

$PS = \sqrt{x^2 + (y-5)^2}$.

By property of parabola

$PS = PM$

$$\frac{x+4y+48}{\sqrt{17}} = \sqrt{x^2+(y-5)^2}.$$

$$\frac{(x+4y+48)^2}{17} = x^2+y^2+25-10y.$$

$$(x+4y)^2+2304+2(x+4y)(48)$$

$$= 17x^2+17y^2+425-170y.$$

$$x^2+16y^2+8xy+2304+96x+384y$$
$$=17x^2+17y^2+425-170y.$$

$$-16x^2-y^2-8xy-96x+\cancel{214}y+2304-425=0$$

$$16x^2+y^2+8xy+96x-554y-1879=0.$$

$\boxed{\text{Ex. 19.1}}$

(6).

~~$m_1 = 0$~~ Slope of axis $m_1 = 0$.

$m_1 m_2 = -1$.

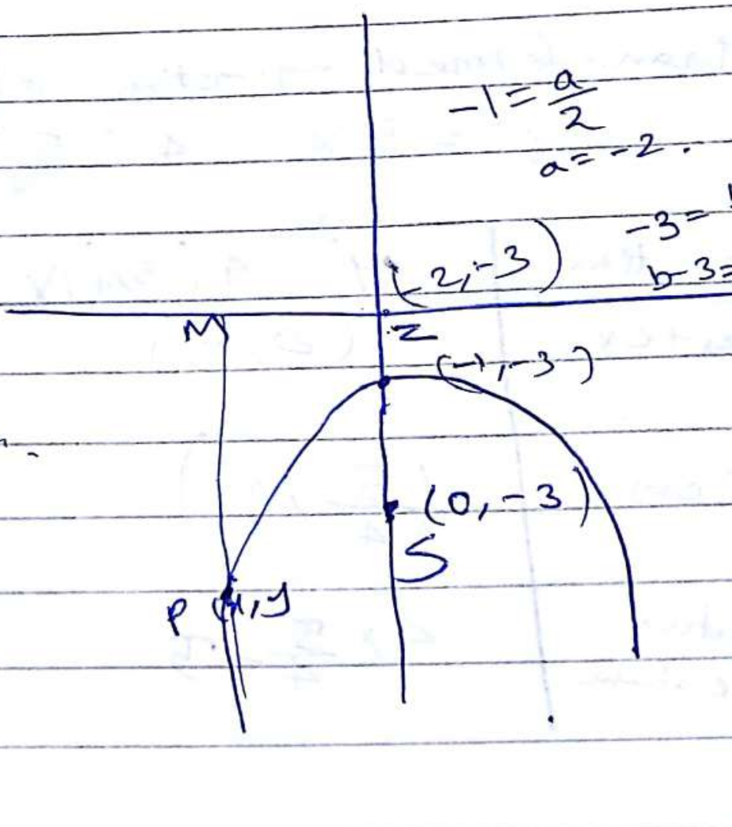
$\therefore$ Slope of m_2: $m_2 = \frac{-1}{0}$

$\cancel{m_2 = \infty}$.

Using point-slope form.

$y+3 = \frac{1}{0}(x+2)$

$x+2 = 0$.



$PS = PM$.

$PM = x+2$.

$PS = \sqrt{x^2+(y+3)^2}$

$\sqrt{x^2+(y+3)^2} = x+2$

$x^2+(y+3)^2 = x^2+4x+4$.

$\cancel{x^2} + y^2+9+6y = \cancel{x^2}+4x+4$

$y^2+6y-4x+5 = 0$.

Q. Find the vertex, focus, latus rectum, axis and directrix of parabola $y^2 = 5x-4y-9$.

$y^2+4y = 5x-9$

$y^2+4y+4 = 5x-9+4$.

$(y+2)^2 = 5x-5$

$= 5(x-1)$

Let $y+2 = Y$ and $x-1 = X$.

Transformed equation of the parabola is.

$Y^2 = 5X = 4\left(\frac{5}{4}\right)X$

$4a = 5$.

$a = 5/4$.

Equation	$Y^2 = 4(5/4)X$	$(y+2)^2 = 5(x-1)$
Vertex	$(0,0)$	$(1,-2)$ $x = X+1 = 0+1 = 1$. $y = Y-2 = 0-2 = -2$
Focus	$\left(\frac{5}{4}, 0\right)$	$\therefore x = X+1 = \frac{5}{4}+1$
Latus Rectum	$4\times\frac{5}{4} = 5$	5.

Axis	$Y = 0.$	$y + 2 = 0$
Directrix	$X + \frac{5}{4} = 0.$	$x - 1 + \frac{5}{4} = 0.$ $4x + 1 = 0$

Q. $x^2 + y = 9x + 14$

$x^2 - 9x = -y - 14$

$x^2 - 9x + \frac{81}{4} = -y - 14 + \frac{81}{4}.$

$\left(x - \frac{9}{2}\right)^2 = -y + \frac{25}{4}$

$\left(x - \frac{9}{2}\right)^2 = -\left(y - \frac{25}{4}\right).$

Let $x - \frac{9}{2} = X$ $\quad Y = y - \frac{25}{4}.$

$X^2 = -Y = -4(1/4$

Equation.		

(11)

Ex. 19.2

$$2x^2 + 5y - 6x = 4$$

$$x^2 + \frac{5}{2}y - 3x = 2$$

$$x^2 - 3x = 2 - \frac{5}{2}y$$

$$x^2 - 3x - \frac{9}{4} + \frac{9}{4} = 2 - \frac{5}{2}y$$

$$\left(x - \frac{3}{2}\right)^2 = 2 + \frac{9}{4} - \frac{5}{2}y$$

$$\left(x - \frac{3}{2}\right)^2 = \frac{17}{4} - \frac{5}{2}y$$

$$\left(x - \frac{3}{2}\right)^2 = -\frac{5}{2}y + \frac{17}{4}$$

$$\left(x - \frac{3}{2}\right)^2 = \cancel{-\frac{5}{2}\left(y + \frac{1}{4_2} \times \frac{2}{5}\right)} = -\frac{5}{2}\left(y - \frac{17}{4_2} \times \frac{2}{5}\right)$$

$$\cancel{= -\frac{5}{2}\left(y + \frac{1}{10}\right)} = -\frac{5}{2}\left(y - \frac{17}{10}\right)$$

$$x - \frac{3}{2} = X \qquad y - \frac{17}{10} = Y \qquad 4A = +\frac{5}{2}$$

$$X^2 = -4AY \qquad A = +\frac{5}{8}$$

Vertex ~~Focus~~ $\left(\frac{3}{2}, \frac{17}{10}\right)$

Focus $= (0, -a)$

$$y - \frac{17}{10} = -\frac{5}{8}$$

$$y = -\frac{5}{8} + \frac{17}{10}$$

$$= \frac{-25 + 68}{40} = \frac{43}{40}$$

$$F = \left(\frac{3}{2}, \frac{43}{40}\right).$$

17×4, 68, 43

Latus Rectum $= 4a = \cancel{4} \times \frac{5}{\cancel{2}} = 10$

axis $= \quad X = 0.$

$x - \frac{3}{2} = 0$

$2x - 3 = 0$

Directrix $= \quad Y - A = 0.$

$y - \frac{17}{10} - \frac{5}{8} = 0.$

$\frac{40y - 68 - 5 \times 5}{40} = 0$

$40y - 68 - 25 = 0.$

$40y - 93 = 0$

Q. $4y^2 + 12x - 12y + 39 = 0.$

$4y^2 - 12y = -12x - 39.$

$y^2 - 3y = -3x - \frac{39}{4}.$

$+\frac{9}{2}$

$y^2 - 3y - \frac{9}{\cancel{4}} + \frac{9}{4} = -3x - \frac{39}{4}.$

$\left(y - \frac{3}{2}\right)^2 = -3x - \frac{39}{4} + \frac{9}{4}$

$\left(y - \frac{3}{2}\right)^2 = -3x - \frac{\overset{15}{\cancel{30}}}{\cancel{4}}$

$= -3\left(x + \frac{\overset{5}{\cancel{15}}}{\cancel{6}\,2}\right)$

$Y^2 = -4AX$

$4A = 3$

$A = \frac{3}{4}$

$Y = y - \frac{3}{2}$

$X = x + \frac{5}{2}.$

Vertex $= (0,0)$

$x + \frac{5}{2} = 0 \qquad y - \frac{3}{2} = 0$

$x = -\frac{5}{2} \qquad y = \frac{3}{2}$

Vertex $\left(-\frac{5}{2}, \frac{3}{2}\right)$

Focus $(-a, 0)$

$\left(-\frac{13}{4}, \frac{3}{2}\right)$

Latus Rectum $= 4a$

$= 3$

Directrix $X - A = 0$

$x + \frac{5}{2} - \frac{3}{4} = 0.$

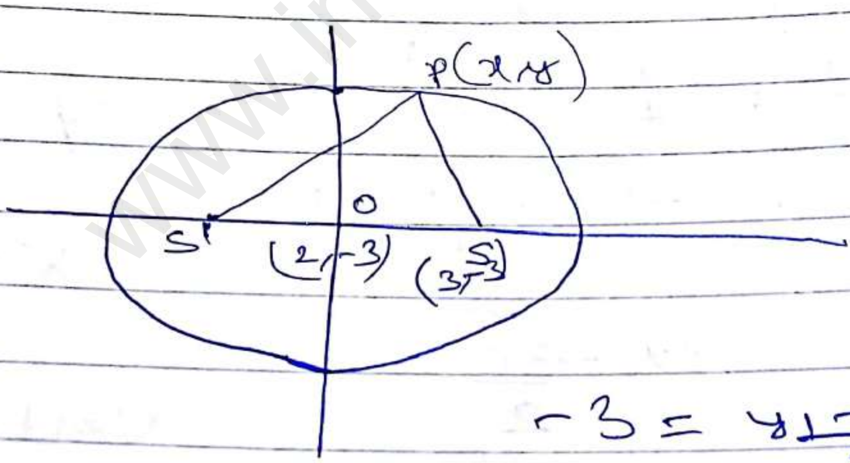
$x + \frac{7}{4} = 0$

$4x + 7 = 0.$

$x + \frac{5}{2} = -\frac{3}{4}$

$x = -\frac{13}{4}$

Q. Find the equation of the ellipse having its centre at the point $(2, -3)$, one focus at $(3, -3)$ and one vertex at $(4, -3)$.



$S'(1, -3)$

$2 = \frac{x_1 + 3}{2}$

$4 = x + 3$

$x = 1$

$-3 = \frac{y_1 - 3}{2}$

$-6 = y_1 - 3$

$y_1 = -6 + 3$

$y = -3$

$3x^2 + 4y^2 - 12x + 24y + 36 = 0.$

$a = 4$

$PS + PS' = 2a$ $\quad$ $2a$ = length of major axis.

Semi-major axis $CA = \sqrt{(4-2)^2 + (-3+3)^2}$

$= \sqrt{4}$

$= 2$

Major axis $= 2 \cdot CA = 2 \times 2 = 4.$

$PS = \sqrt{(x-3)^2 + (y+3)^2}.$

$PS' = \sqrt{(x-1)^2 + (y+3)^2}$

$\sqrt{(x-3)^2 + (y+3)^2} + \sqrt{(x-1)^2 + (y+3)^2} = 8.$

Q. Find the eccentricity, foci and the length of the latus rectum of the ellipse

$9x^2 + 16y^2 = 144$

$\frac{9x^2}{144/16} + \frac{16y^2}{144/9} = \frac{144}{144}$

$\frac{x^2}{16} + \frac{y^2}{9} = 1$

$a = 4$ $b = 3$

$b^2 = a^2(1-e^2)$

$9 = 16 - 16e^2$

$16e^2 = 7$

$e^2 = \frac{7}{16}$

$e = \frac{\sqrt{7}}{4}$

foci $= (\pm ae, 0)$

$= (\pm \frac{\sqrt{7}}{4} \times 4, 0)$

$= (\pm \sqrt{7}, 0)$

Latus rectum $= \frac{2b^2}{a}$

$= 2 \times \frac{9}{4_2} = \frac{9}{2}$

(b) $5x^2 + 4y^2 = 1$

~~$\frac{x^2}{4} + \frac{y^2}{5} = \frac{1}{20}$~~

$\frac{x^2}{1/5} + \frac{y^2}{1/4} = 1$

~~$= \frac{1}{\sqrt{5}}$ $b = \frac{1}{2}$ foci =~~

$b^2 = a^2(1-e^2)$

~~$\frac{1}{4} = \frac{1}{5}(1-e^2)$~~

$\frac{1}{4} = \frac{1}{5} - \frac{1}{5}e^2$

~~$\frac{1}{5}e^2 = \frac{1}{5} - \frac{1}{4}$~~

$\frac{1}{5}e^2 = \frac{4-5}{20}$

~~$e^2 = \frac{1}{20} \times 5$~~

$\frac{1}{4}$

$e = 1$

Q. $3x^2 + 2y^2 = 6$

$\frac{x^2}{2} + \frac{y^2}{3} = 1$

$a^2 = 3 \qquad b^2 = 2$

Q. Find the eccentricity of an ellipse of its latus rectum is

(a) half its major axis.

(b) half its minor axis.

$LR = \frac{1}{2}$ major axis.

$\frac{2b^2}{a} = \frac{1}{2} \cdot 2a$.

$a^2 = 2b^2$.

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Q. Find the eccentricity, foci, centre, latus rectum and lengths of axes of the ellipse $3x^2+4y^2-12x-8y+4=0$.

$$3x^2-12x+4y^2-8y+4=0.$$

$$3(x^2-4x)+4(y^2-2y)+4=0$$

$$3(x^2-4x+4-4)+4(y^2-2y+1-1)+4=0$$

$$3\left[(x-2)^2-4\right]+4\left[(y-1)^2-1\right]+4=0$$

$$3(x-2)^2-12+4(y-1)^2-\cancel{4}+\cancel{4}=0$$

$$3(x-2)^2+4(y-1)^2=12.$$

$$\frac{(x-2)^2}{4}+\frac{(y-1)^2}{3}=1$$

$x-2=X$ $\quad$ $y-1=X$

$x=X+2$ $\quad$ $y=X+1$ $\quad$ Foci $=$

Centre $(2,1)$

$$e=\sqrt{1-\frac{3}{4}}=\sqrt{\frac{1}{4}}=\frac{1}{2}.$$

Latus rectum $=\dfrac{2b^2}{a}=\dfrac{2\times3}{2}=3$

$\sqrt{1-\frac{b^2}{a^2}}$

Foci

$$\frac{X^2}{4}+\frac{Y^2}{3}=1 \qquad \frac{(x-2)^2}{4}+\frac{(y-1)^2}{3}=1$$

$(\pm1,0)$ $\qquad$ $(3,1)$

$(1,1)$

$L.R = \frac{2b^2}{a}$
$= \frac{2 \times \frac{9}{4}}{3} = \frac{3}{2}$

Latus rectum	$\frac{2 \times 3}{2} = 3$	3
Length of the axes	$2 \times 2 = 4$ $2\sqrt{3} = \sqrt{3}$	4 $2\sqrt{3}$

Eq^n of directrix, $\left(X = \frac{\pm a}{e} \right)$

Q. (4) Ex. 19.4

$$x^2 + 4y^2 - 4x + 24y + 31 = 0.$$
$$x^2 - 4x + 4y^2 + 24y + 31 = 0$$
$$x^2 - 4x + 4 - 4 + 4(y^2 + 6y + 9 - 9) + 31 = 0.$$
$$(x-2)^2 - 4 + 4\left[(y+3)^2 - 9\right] + 31 = 0$$
$$(x-2)^2 - 4 + 4(y+3)^2 - 36 + 31 = 0.$$
$$(x-2)^2 + 4(y+3)^2 - 5 - 4 = 0.$$
$$(x-2)^2 + 4(y+3)^2 = 9$$
$$\frac{(x-2)^2}{9} + \frac{(y+3)^2}{9/4} = 1$$

foci $= ae$
$= 3 \times \frac{\sqrt{3}}{2}$
$\frac{3\sqrt{3}}{2}$

$e = \sqrt{1 -$
$= \sqrt{}$

	$\frac{X^2}{9} + \frac{Y^2}{9/4} = 1$	$\frac{(x-2)^2}{9} + \frac{(y+3)^2}{9/4}$
Centre	$(0, 0)$	$(2, -3)$
Foci		$\left(2 \pm \frac{3\sqrt{3}}{2}, -3\right.$
length of axes		6
Latus rectum		$3/2$

$(y+3)^2$
y^2+9+6y

Q $x^2+2y^2-2x+12y+10=0$

$x^2-2x+2y^2+12y+10=0.$

$x^2-2x+1-1+2(y^2+6y+9-9)+10=0$

$(x-1)^2-1+2\left[(y+3)^2-9\right]+10=0$

$(x-1)^2+2(y+3)^2-18-1+10=0.$

$(x-1)^2+2(y+3)^2=9$

$\frac{(x-1)^2}{9}+\frac{(y+3)^2}{9/2}=1$

$e=\sqrt{1-\frac{b^2}{a^2}}$

$=\sqrt{1-\frac{9/2}{9}}$ Centre

$\frac{9}{2}\times\frac{1}{9}$

$=\sqrt{1-\frac{1}{2}}=\frac{1}{\sqrt{2}}$

$e=\sqrt{1-\frac{b^2}{a^2}}$

$=\sqrt{1-\frac{9}{4}\times\frac{1}{9}}$

$=\sqrt{1-\frac{1}{4}}$

$=\frac{\sqrt{7}}{4}$

$=\sqrt{\frac{3}{4}}=\frac{\sqrt{3}}{2}$

Latus rectum $=\frac{2b^2}{a}=\frac{2\times\frac{9}{2}}{3}=3$

Focus	$=\left(\pm 3\times\frac{1}{\sqrt{2}}, 0\right)$ $x=\pm\frac{3}{\sqrt{2}}$
~~Fo~~	$x-1=\pm\frac{3}{\sqrt{2}}.$
Foci	$\left(x=1\pm\frac{3}{\sqrt{2}}, -3\right)$
Axes	$2\times3=6$
Centre	$(1,-3)$

$1\times\frac{4}{8}$

6^2

3 | 63
3 | 21
  | 7

The

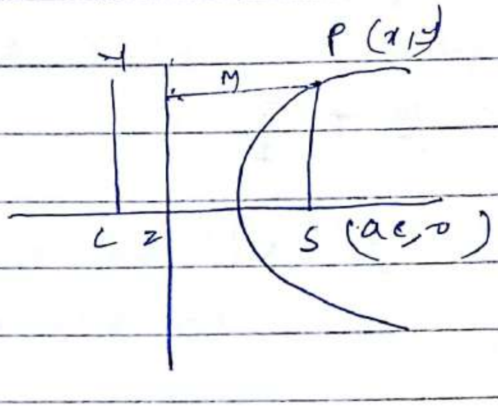
L

$b^2 = a^2(e^2-1) \quad e>1$

Centre $(0,0)$

Foci $(\pm ae, 0)$

Latus rectum $= \frac{2b^2}{a}$



$SS' = 2ae$.

length of the axes $= 2a$

$= 2b$.

Directrix, $x = \pm \frac{a}{e}$

$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Foci $= (0, \pm ae)$

Directrices $y = \pm a/e$

Q. Show that the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ have same foci

$e = \sqrt{1 - \frac{9}{16}}$

$= \sqrt{\frac{7}{16}}$

$= \frac{\sqrt{7}}{4}$

foci $= (\pm ae, 0)$

$= (\pm 4 \times \frac{\sqrt{7}}{4}, 0)$

$= (\pm\sqrt{7}, 0)$

$\frac{x^2}{144/25} - \frac{y^2}{81/25} = 1$

$e = \sqrt{1 - \frac{81}{25} \times \frac{25}{144}}$

$= \sqrt{\frac{63}{144}}$

$= \frac{3\sqrt{7}}{12}$

foci $\left(\pm \frac{12}{5} \times \frac{\sqrt{7}}{4}\right.$

$$e = \sqrt{1-\frac{7}{16}}$$

$$= \sqrt{\frac{9}{16}}$$

$$= \frac{3}{4}$$

foci $\left(\pm \cancel{4}\times\frac{3}{\cancel{4}}, 0\right)$

$(\pm 3, 0)$

$$\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$$

$$\frac{x^2}{144/25} - \frac{y^2}{81/25} = 1$$

$$e = \sqrt{1+\frac{81}{\cancel{25}}\times\frac{\cancel{25}}{144}}$$

$144 + 81 = 225$

$$= \sqrt{\frac{225}{144}}$$

$$= \frac{15}{12}$$

foci $\left(\pm \frac{\cancel{12}}{8}\times\frac{\cancel{15}^{3}}{\cancel{12}}, 0\right)$

foci $(\pm 3, 0)$

Ex. 19.5

Q. (2) Find the equation of the hyperbola whose focus is $(1,1)$, directrix is $3x+4y+8=0$ and $e=2$.

$PS = e\, PM$.

$PS = 2 \cdot PM$.

$$\sqrt{(x-1)^2+(y-1)^2} = 2\left(\frac{3x+4y+8}{\sqrt{9+16}}\right)$$

$$(x-1)^2+(y-1)^2 = 4\left[\frac{(3x+4y+8)^2}{25}\right]$$

$$\begin{array}{r}514\\25\\\hline 39\end{array} \qquad \begin{array}{r}192\\50\\\hline 242\end{array} \qquad \begin{array}{r}256\\50\\\hline 306\end{array}$$

$$(x-1)^2+(y-1)^2 = 4\left[\frac{(3x+4y)^2+64+16(3x+4y)}{25}\right]$$

$$(x-1)^2+(y-1)^2 = \frac{4}{25}\left[9x^2+16y^2+24xy+64+48x+64y\right]$$

$$25\left(x^2+1-2x+y^2+1-2y\right) = 4\left(9x^2+16y^2+24xy+64+48x+64y\right)$$

$$25x^2+25-50x+25y^2+25-50y = 36x^2+64y^2+96xy+256+192x+256y$$

$$-11x^2-39y^2-242x-306y-96xy-206=0.$$

$$11x^2+39y^2+242x+306y+96xy+206=0$$

Q. Find the eccentricity, centre, foci, latus rectum length of the axes and directrices of the hyperbola $9x^2-16y^2-18x+32y-151=0$

Sol:-
$$9x^2-18x-16y^2+32y-151=0.$$
$$9(x^2-2x)-16(y^2-2y)-151=0$$
$$9(x^2-2x+1-1)-16(y^2-2y+1-1)-151=0$$
$$9\left[(x-1)^2-1\right]-16\left[(y-1)^2-1\right]-151=0$$
$$9(x-1)^2-9-16(y-1)^2+16-151=0.$$
$$9(x-1)^2-16(y-1)^2-144=0$$
$$\frac{9(x-1)^2}{144}-\frac{16(y-1)^2}{144}=1$$

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$4\,a\,e$ $\frac{5}{4} \times 4$

26th Au

Q.

$$\frac{(x-1)^2}{16} - \frac{(y-1)^2}{9} = 1$$

$a^2 = 16$, $b^2 = 9$

$e = \sqrt{1+\frac{9}{16}}$

$= \sqrt{\frac{25}{16}}$

$= \frac{5}{4}$

Centre $(1,1)$

Foci $(\pm 5, 0)$

$x-1 = \pm 5$ (+) $y-1=0$

$x-1 = 5$ $(y=1)$

$(x=6)$ $(6, 1)$

$x-1 = -5$

$(x = -4)$ $(-4, 1)$

Lengths of the axes $= 8$

$= 6$

Latus rectum $= \frac{2b^2}{a}$

$= \frac{\cancel{2} \times 9}{\cancel{4}\, 2} = \frac{9}{2}$.

Directrix $x-1 = \pm\left(\frac{4}{\cancel{8}4} \times \frac{4}{5}\right)$

$9 \div \frac{5}{4}$

$x-1 = +\frac{16}{5}$ $x-1 = -\frac{16}{5}$

$5x - 5 = 16$ $5x - 5 = -16$

$5x - 5 - 16 = 0$ $5x - 5 + 16 = 0$

$5x - 21 = 0$ $5x + 11 = 0$

26th Aug 2015

$$\begin{array}{r} 164 \\ 16 \\ \hline 180 \\ 36 \\ \hline 144 \end{array}$$

Q. Find eccentricity, centre, foci, latus rectum and directrices of the hyperbola

$$16x^2 - 9y^2 + 32x + 36y - 164 = 0$$

$$16x^2 + 32x - 9y^2 + 36y - 164 = 0.$$

$$16(x^2 + 2x) - 9(y^2 - 4y) - 164 = 0.$$

$$16[x^2 + 2x + 1 - 1] - 9[y^2 - 4y + 4 - 4] - 164 = 0.$$

$$16[(x+1)^2 - 1] - 9[(y-2)^2 - 4] - 164 = 0.$$

$$16[(x+1)^2] - 16 - 9(y-2)^2 + 36 - 164 = 0.$$

$$16(x+1)^2 - 9(y-2)^2 = 164 - 36 + 16$$

$$= 144.$$

$$16(x+1)^2 - 9(y-2)^2 = 144$$

$$\frac{\cancel{16}(x+1)^2}{\cancel{144}_{9}} - \frac{\cancel{9}(y-2)^2}{\cancel{144}_{16}} = 1$$

$$\frac{(x+1)^2}{9} - \frac{(y-2)^2}{16} = 1$$

$a^2 = 9 \qquad b^2 = 16.$

$$e = \sqrt{1 + \frac{16}{9}} = \sqrt{\frac{25}{9}} = \frac{5}{3}$$

$\boxed{\text{Centre } (-1, 2)}$

Foci $\left(\pm \cancel{3} \times \frac{5}{\cancel{3}}, 0\right)$

$(\pm 5, 0)$

$x + 1 = \pm 5 \qquad x = -6$

$x = 5 - 1$

$x = 4$

$y - 2 = 0$

$y = 2.$

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$\boxed{\text{Foci} \quad (4,2) \text{ and } (-6,2)}$

$$\text{Latus rectum} = \frac{2b^2}{a}$$

$$= \frac{2 \times 16}{3}$$

$$\boxed{= \frac{32}{3}}$$

Directrix $\quad x+1 = \pm \frac{3}{} \times \frac{3}{5}$

$$x+1 = \pm \frac{9}{5}$$

$\cancel{x+1} \quad \cancel{x = \frac{9}{5} - 1}$

$\cancel{x = \frac{4}{5}}$

$5x+5 = +9$

$\boxed{5x - 4 = 0}$

$5x+5 = -9$

$\boxed{5x + 14 = 0}$

$I\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Matrix

A system of mn numbers arranged along m - rows and n-columns is called an m by n (m×n) matrix. A matrix is denoted by a single capital letter.

Thus,

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & & & & \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}$$

Square matrix If $m=n$.

$$A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$$ is a square matrix of order 3

Singular matrix: A square matrix is called singular if $|A|=0$.

Non-singular if $|A| \neq 0$.

Identity (Unit matrix)

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ all primary diagonal elements are 1 and other are zero

$a_{mn} = 0$ if $m \neq n$
$= 1$ if $m=n$

27/8/15

Addition.

If A and B are two matrices of the same order, then addition is defined.

Let $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$ and $B = \begin{bmatrix} e_1 & e_2 & e_3 \\ f_1 & f_2 & f_3 \\ g_1 & g_2 & g_3 \end{bmatrix}$

$$A + B = \begin{bmatrix} a_1+e_1 & a_2+e_2 & a_3+e_3 \\ b_1+f_1 & b_2+f_2 & b_3+f_3 \\ c_1+g_1 & c_2+g_2 & c_3+g_3 \end{bmatrix}$$

$$A - B = \begin{bmatrix} a_1-e_1 & a_2-e_2 & a_3-e_3 \\ b_1-f_1 & b_2-f_2 & b_3-f_3 \\ c_1-g_1 & c_2-g_2 & c_3-g_3 \end{bmatrix}$$

Addition is commutative $A+B = B+A$

Subtraction is not commutative

$A - B \neq B - A$

Scalar Multiplication

$$KA = K \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$$

$$= \begin{bmatrix} ka_1 & ka_2 & ka_3 \\ kb_1 & kb_2 & kb_3 \\ kc_1 & kc_2 & kc_3 \end{bmatrix}$$

Multiplication :- If A and B are two matrices such that no. of columns of A is equal to rows of B, then AB is defined.

Multiplication is not commutative $AB \neq BA$

Let $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$ $\quad B = \begin{bmatrix} d_1 & d_2 & d_3 \\ e_1 & e_2 & e_3 \\ f_1 & f_2 & f_3 \end{bmatrix}$

$$AB = \begin{bmatrix} a_1d_1 + a_2e_1 + a_3f_1 & a_1d_2 + a_2e_2 + a_3f_2 & a_1d_3 + a_2e_3 + a_3f_3 \\ b_1d_1 + b_2e_1 + b_3f_1 & b_1d_2 + b_2e_2 + b_3f_3 & b_1d_3 + b_2e_3 + b_3f_3 \\ c_1d_1 + c_2e_1 + c_3f_1 & c_1d_2 + c_2e_2 + c_3f_3 & c_1d_3 + c_2e_3 + c_3f_3 \end{bmatrix}$$

Q. If $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 6 \\ 3 & 4 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 2 \\ 5 & 0 & 1 \end{bmatrix}$

Compute AB and BA and prove that $AB \neq BA$

$$AB = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 6 \\ 3 & 4 & 7 \end{bmatrix} \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 2 \\ 5 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1(3) + 8 + 0 & 2 + 2 + 0 & 1 + 4 + 0 \\ 6 + 0 + 30 & 4 + 0 + 0 & 2 + 0 + 6 \\ 9 + 16 + 35 & 6 + 4 + 0 & 3 + 8 + 7 \end{bmatrix} = \begin{bmatrix} 11 & 4 & 5 \\ 36 & 4 & 8 \\ 60 & 10 & 18 \end{bmatrix}$$

$$BA = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 2 \\ 5 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 6 \\ 3 & 4 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 3+4+3 & 6+0+4 & 0+12+7 \\ 4+2+6 & 8+0+8 & 0+6+14 \\ 5+0+3 & 10+0+4 & 0+0+7 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 10 & 19 \\ 12 & 16 & 20 \\ 8 & 14 & 7 \end{bmatrix}$$

Hence $AB \neq BA$. Proved

Q. Find $A^2 + 2A + 3I$ if I = is an unit matrix of order 3 and

$$A = \begin{bmatrix} 2 & 1 & -3 \\ 1 & 0 & 5 \\ -2 & 4 & 1 \end{bmatrix}$$

$$A \cdot A = \begin{bmatrix} 2 & 1 & -3 \\ 1 & 0 & 5 \\ -2 & 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & -3 \\ 1 & 0 & 5 \\ -2 & 4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4+1+6 & 2+0-12 & -6+5-3 \\ 2+0-10 & 1+0+20 & -3+0+5 \\ -4+4-2 & -2+0+4 & 6+20+1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 11 & -10 & -4 \\ -8 & 21 & 2 \\ -2 & 2 & 27 \end{bmatrix}$$

$$2A = \begin{bmatrix} 4 & 2 & -6 \\ 2 & 0 & 10 \\ -4 & 8 & 2 \end{bmatrix} \qquad 3I = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$2A + 3I = \begin{bmatrix} 7 & 2 & -6 \\ 2 & 3 & 10 \\ -4 & 8 & 5 \end{bmatrix}$$

$$A^2 + 2A + 3I = \begin{bmatrix} 11 & -10 & -4 \\ -8 & 21 & 2 \\ -2 & 2 & 27 \end{bmatrix} + \begin{bmatrix} 7 & 2 & -6 \\ 2 & 3 & 10 \\ -4 & 8 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 18 & -8 & -10 \\ -6 & 24 & 12 \\ -6 & 10 & 32 \end{bmatrix}$$

$$A \cdot A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1-4-9 & -2-6+3 & 3+4+6 \\ 2+6+3 & -4+9-1 & 6-3-2 \\ -3+2+6 & +6+3+2 & -9-1+4 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 6 & -5 & 13 \\ 5 & 4 & 1 \\ 11 & -1 & 12 \end{bmatrix}$$

$$3A = \begin{bmatrix} 3 & -6 & 9 \\ 6 & 9 & -3 \\ 9 & 3 & 6 \end{bmatrix} \qquad 6I \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$A^2 - 3A = \begin{bmatrix} 3 & 1 & 4 \\ -1 & -5 & 4 \\ 2 & -14 & 6 \end{bmatrix}$$

$$A^2 - 3A + 6I = \begin{bmatrix} 3 & 1 & 4 \\ -1 & -5 & 4 \\ 2 & -4 & 6 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} -12 & -5 & 13 \\ 11 & 4 & 1 \\ -7 & 11 & -6 \end{bmatrix}$$

$$3A = \begin{bmatrix} 3 & -6 & 9 \\ 6 & 9 & -3 \\ -9 & 3 & 6 \end{bmatrix}$$

$$A^2 - 3A = \begin{bmatrix} -15 & 1 & 4 \\ 5 & -5 & 4 \\ 2 & 8 & -12 \end{bmatrix}$$

$$A^2 - 3A + 6I = \begin{bmatrix} -15 & 1 & 4 \\ 5 & -5 & 4 \\ 2 & 8 & -12 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} -9 & 1 & 4 \\ 5 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -25 & 5 & 20 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2 & -1 & -3 \\ 1 & 2 & -4 \\ -3 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & -1 & -3 \\ 1 & 2 & -4 \\ -3 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 4-1+9 & -2-2+3 & -6+4+0 \\ 2+2+12 & -1+4+4 & -3-8+0 \\ -6-1+0 & 3-2+0 & 9+4+0 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & -1 & -2 \\ 16 & 7 & -11 \\ -7 & 1 & 13 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 12 & -1 & -2 \\ 16 & 7 & -11 \\ -7 & 1 & 13 \end{bmatrix} \Big[$$

Q. If $A = \begin{bmatrix} 3 & 2 & 1 \\ -1 & 2 & 0 \\ 1 & 2 & 5 \end{bmatrix}$ $B = \begin{bmatrix} 2 & -1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$ $C = \begin{bmatrix} -3 & 2 \\ 2 & 1 \\ -1 & 3 \end{bmatrix}$

$$B + C = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 0 & 3 \end{bmatrix}$$

$$A(B+C) = \begin{bmatrix} 3 & 2 & 1 \\ -1 & 2 & 0 \\ 1 & 2 & 5 \end{bmatrix}_{3\times 3} \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 0 & 3 \end{bmatrix}_{3\times 2}$$

$$= \begin{bmatrix} -3+4+0 & 3+4+3 \\ 1+4+0 & -1+4+0 \\ -1+4+0 & 1+4+15 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & -1 \\ -2 & 3 \\ 7 & -1 \end{bmatrix}$$

$$AC = \begin{bmatrix} 3 & 2 & \\ -1 & 2 & 0 \\ 1 & 2 & 5 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ 2 & 1 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -9+4-1 & 6+2+3 \\ 3+4+0 & -2+2+0 \\ -3+4-5 & 2+2+15 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & 11 \\ 7 & 0 \\ -4 & 19 \end{bmatrix}$$

$$AB+BC = \begin{bmatrix} 7 & -1 \\ -2 & 3 \\ 7 & +1 \end{bmatrix} \begin{bmatrix} -6 & 11 \\ 7 & 0 \\ -4 & 19 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 10 \\ 5 & 3 \\ 3 & 20 \end{bmatrix}$$

$$= A(B+C)$$ Hence verified

Q. (i) For the following matrices

$$A = \begin{bmatrix} 2 & 3 & -1 \\ 3 & 0 & 2 \end{bmatrix}_{2\times3}, \quad B = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}_{3\times1} \quad C = \begin{bmatrix} 1 & -2 \end{bmatrix}_{1\times2}$$

~~show~~ Prove that $A(BC) = (AB)C$.

$$AB = \begin{bmatrix} 2-3-2 \\ 3+0+4 \end{bmatrix}_{2\times1} = \begin{bmatrix} -3 \\ 7 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & -2 \end{bmatrix}$$

$$BC = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ -1 & 2 \\ 2 & -4 \end{bmatrix}$$

$2-3-2$

$3+4$ $\quad -4+6+4$

$-6+0-8$

$$A(BC) = \begin{bmatrix} 2 & 3 & -1 \\ 2 & 0 & 2 \end{bmatrix}_{2\times 3} \begin{bmatrix} 1 & -2 \\ -1 & 2 \\ 2 & -4 \end{bmatrix}_{3\times 2}$$

$$= \begin{bmatrix} -3 & 6 \\ 7 & -14 \end{bmatrix}_{2\times 2}$$

$$(AB)C = \begin{bmatrix} -3 \\ 7 \end{bmatrix}_{2\times 1} \begin{bmatrix} 1 & -2 \end{bmatrix}_{1\times 2}$$

$$= \begin{bmatrix} -3 & +6 \\ 7 & -14 \end{bmatrix}_{2\times 2}.$$

Hence Proved

(2) If $A = \begin{bmatrix} 2 & 3 & -1 \\ 3 & 2 & 0 \\ 1 & -2 & -3 \end{bmatrix}_{3\times 3}$ and $B = \begin{bmatrix} -1 & 2 \\ 2 & 5 \\ 0 & 1 \end{bmatrix}_{3\times 2}$.

Prove that $(AB)' = B'A'$

$$AB = \begin{bmatrix} -2+6+0 & 4+15-1 \\ -3+4+0 & 6+10+0 \\ -1-4+0 & 2-10-3 \end{bmatrix}_{3\times 2} = \begin{bmatrix} 4 & 18 \\ 1 & 16 \\ -5 & -11 \end{bmatrix}$$

$$(AB)' = \begin{bmatrix} 4 & 1 & -5 \\ 18 & 16 & -11 \end{bmatrix}$$

$$A' = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 2 & -2 \\ -1 & 0 & -3 \end{bmatrix}_{3\times3} \qquad B' = \begin{bmatrix} -1 & 2 & 0 \\ 2 & 5 & 1 \end{bmatrix}_{2\times3}$$

$$B'A' = \begin{bmatrix} -1 & 2 & 0 \\ 2 & 5 & 1 \end{bmatrix}_{2\times3} \begin{bmatrix} 2 & 3 & 1 \\ 3 & 2 & -2 \\ -1 & 0 & -3 \end{bmatrix}_{3\times3}$$

$$= \begin{bmatrix} -2+6+0 & -3+4+0 & -1-4+0 \\ +4+15-1 & 6+10+0 & 2-10-\cancel{13}\,3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 1 & -5 \\ 18 & 16 & -11 \end{bmatrix}$$

$(AB)' = B'A'$ Hence Proved

$\boxed{\text{Minors}}$

Let $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$ $\qquad \cancel{a_1}(b_2c_3 - c_2b_3)$

Minor of $a_1 = \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix}$

Minor of $b_1 = \begin{vmatrix} a_2 & a_3 \\ c_2 & c_3 \end{vmatrix}$

Minor of $c_2 = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix}$

$\boxed{\text{Cofactor} = (-1)^{m+n} \cdot \text{minor}}$ $m \rightarrow$ no. of rows
$n \rightarrow$ no. of columns.

Cofactor of $a_1 = (-1)^{1+1} \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} = \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix}$

Cofactor of $b_1 = (-1)^{2+1} \begin{vmatrix} a_2 & a_3 \\ c_2 & c_3 \end{vmatrix} = -\begin{vmatrix} a_2 & a_3 \\ c_2 & c_3 \end{vmatrix}$.

Cofactor of $c_2 = (-1)^{3+2} \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} = -\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix}$

Adjoint of a matrix

Let $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$

Matrix of Cofactors $= \begin{bmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{bmatrix}$

Transpose of matrix of cofactors $\begin{bmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_2 & C_3 \end{bmatrix}$

is called adjoint of matrix A.

It is denoted by adj A or Adj A.

$$\text{adj}A = \begin{bmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_2 & C_3 \end{bmatrix}$$

Property $A(\text{adj}A) = (\text{adj}A).A$
$= |A| I$.

Q. Find the adjoint of the matrix

$$A = \begin{bmatrix} 1 & 4 & 5 \\ 3 & 2 & 6 \\ 0 & 1 & 0 \end{bmatrix}$$

$AB \neq BA$

and prove that $A(\text{adj}A) = (\text{adj}A)A = |A|I$

$C_{11} = (-1)^2 (0-6) = -6$
$C_{12} = (-1)^3 (0) = 0$
$C_{13} = (-1)^4 (3-0) = 3$
$C_{21} = (-1)^3 (0-5) = 5$
$C_{22} = (-1)^4 (0) = 0$
$C_{23} = (-1)^5 (1) = -1$

$C_{31} = (-1)^4 \cancel{(6-15)}\ (24-10) = 14$

$C_{32} = (-1)^5 \ (6-15) = 9$

$C_{33} = (-1)^6 \ (2-12) = -10$

Matrix of cofactors $= \begin{bmatrix} -6 & 0 & 3 \\ 5 & 0 & -1 \\ 14 & 9 & -10 \end{bmatrix}$

$\text{Adj } A = \begin{bmatrix} -6 & 5 & 14 \\ 0 & 0 & 9 \\ 3 & -1 & -10 \end{bmatrix}$

$A(\text{adj}A) = \begin{bmatrix} 1 & 4 & 5 \\ 3 & 2 & 6 \\ 0 & 1 & 6 \end{bmatrix} \begin{bmatrix} -6 & 5 & 14 \\ 0 & 0 & 9 \\ 3 & -1 & -10 \end{bmatrix}$

$= \begin{bmatrix} -6+0+15 & 5+0-5 & 14+36-50 \\ -18+0+18 & 15+0-6 & 42+18-60 \\ 0+0+18 & 0+0-6 & 0+9-60 \end{bmatrix}$

$= \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 18 & -6 & -51 \end{bmatrix}$

~~(adj A).A =~~ [

$$(\text{adj}A)\cdot A = \begin{bmatrix} -6 & 5 & 14 \\ 0 & 0 & 9 \\ 3 & -1 & -10 \end{bmatrix} \begin{bmatrix} 1 & 4 & 5 \\ 3 & 2 & 6 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -6+15+0 & -24+10+14 & -30+30+0 \\ 0+0+0 & 0+0+9 & 0+0+0 \\ 3-3+0 & 12-2-10 & 15-6+0 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$|A| = 1(-6) + 4(0) + 5(3)$$
$$= -6 - 0 + 15$$
$$= 9$$

$$|A|\cdot I = 9 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$A(\cdot \text{adj}A) = (\text{adj}A)\cdot A = |A|\cdot I$$

Hence Proved

Q. Find the adjoint of the matrix.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 1 & 4 & 3 \end{bmatrix}$$

and prove that $A\cdot(\text{adj}A) = (\text{adj}A)\cdot A = |A|I$

$C_{11} = (-1)^2 \ (9-16) = -7$
$C_{12} = (-1)^3 \ (3-4) = +1$
$C_{13} = (-1)^4 \ (4-3) = +1$
$C_{21} = (-1)^3 \ (6-12) = +6$
$C_{22} = (-1)^4 \ (3-3) = 0$
$C_{23} = (-1)^5 \ (4-2) = -2$
$C_{31} = (-1)^4 \ (8-9) = -1$
$C_{32} = (-1)^5 \ (4-3) = -1$
$C_{33} = (-1)^6 \ (3-2) = +1$

$$\text{Matrix of cofactors} = \begin{bmatrix} -7 & 1 & 1 \\ 6 & 0 & -2 \\ -1 & -1 & 1 \end{bmatrix}$$

$$\text{Adj}A = \begin{bmatrix} -7 & 6 & -1 \\ 1 & 0 & -1 \\ 1 & -2 & 1 \end{bmatrix}$$

$$A \cdot \text{adj}A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 1 & 4 & 3 \end{bmatrix} \begin{bmatrix} -7 & 6 & -1 \\ 1 & 0 & -1 \\ 1 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -7+2+3 & 6+0-6 & -1-2+3 \\ -7+3+4 & 6+0-8 & -1-3+4 \\ -7+4+3 & 6+0-6 & -1-4+3 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

$$(\operatorname{adj} A) \cdot A = \begin{bmatrix} -7 & 6 & -1 \\ 1 & 0 & -1 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 1 & 4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -7+6-1 & -14+18-4 & -21+24-3 \\ 1+0-1 & 2+0-4 & 3+0-3 \\ 1-2+1 & 2-6+4 & 3-8+3 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

$$|A| = 1(9-16) - 2(3-4) + 3(4-3)$$
$$= -7 + 2 + 3$$
$$= -5 + 3$$
$$= -2$$

$$|A| \cdot I = -2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

$$A \cdot (\operatorname{adj} A) = (\operatorname{adj} A) \cdot A = |A| \cdot I$$

3/8/15

Inverse of Matrix

If A and B are two square matrices of the same order such that

$$AB = BA = I.$$

then B is called inverse of A i.e $B = A^{-1}$ and A is called inverse of B i.e $A = B^{-1}$

$A \,\mathrm{adj}A = |A|\, I$ $\qquad A \cdot A^{-1} = I.$

$= |A| \cdot A \cdot A^{-1}.$

$$A^{-1} = \frac{1}{|A|}\,\mathrm{adj}A \quad \text{if } |A| \neq 0$$

Q. Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

$$C_{11} = (-1)^2 \begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix} = (+1)(3-1) = 2$$

$$C_{12} = (-1)^3 \begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} = (-1)(2+1) = -3$$

$$C_{13} = (-1)^4 \begin{vmatrix} 2 & 3 \\ -1 & 1 \end{vmatrix} = (+1)(2+3) = 5$$

$$C_{21} = (-1)^3 \begin{vmatrix} 2 & 5 \\ 1 & 1 \end{vmatrix} = (-1)(2-5) = 3$$

$$C_{22} = (-1)^4 \begin{vmatrix} 1 & 5 \\ -1 & 1 \end{vmatrix} = (+1)(1+5) = 6$$

$$C_{23} = (-1)^5 \begin{vmatrix} 1 & 2 \\ -1 & 1 \end{vmatrix} = (-1)(1+2) = -3$$

$$C_{31} = (-1)^4 \begin{vmatrix} 2 & 5 \\ 3 & 1 \end{vmatrix} = (+1)(2-15) = -13$$

$$C_{32} = (-1)^5 \begin{vmatrix} 1 & 5 \\ 2 & 1 \end{vmatrix} = (-1)(1-10) = 9$$

$$C_{33} = (-1)^6 \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = (+1)(3-4) = -1$$

$$\text{Matrix of cofactors} = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 6 & -3 \\ -13 & 9 & -1 \end{bmatrix}$$

$$\text{Adjoint} = \begin{bmatrix} 2 & 3 & -13 \\ -3 & 6 & 9 \\ 5 & -3 & -1 \end{bmatrix}$$

$$|A| = 1(2) - 2(2+1) + 5(2+3)$$
$$= 2 - 6 + 25$$
$$= -4 + 25$$
$$= 21$$

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

$$= \frac{1}{21}\begin{bmatrix} 2 & 3 & -13 \\ -3 & 6 & 9 \\ 5 & -3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{21} & \frac{\cancel{3}\,1}{7} & -\frac{13}{21} \\ -\frac{1}{7} & \frac{\cancel{6}\,2}{\cancel{21}\,7} & \frac{\cancel{9}\,3}{\cancel{21}\,7} \\ \frac{5}{21} & -\frac{\cancel{3}}{\cancel{21}\,7} & -\frac{1}{21} \end{bmatrix}$$

$$= \begin{bmatrix} 2/21 & 1/7 & -13/21 \\ -1/7 & 2/7 & 3/7 \\ 5/21 & -1/7 & -1/21 \end{bmatrix}$$

$$A \cdot A^{-1} = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2/21 & 1/7 & -13/21 \\ -1/7 & 2/7 & 3/7 \\ 5/21 & -1/7 & -1/21 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{21} - \frac{2}{7} + \frac{25}{21} & \frac{1}{7} + \frac{4}{7} - \frac{5}{7} & -\frac{13}{21} + \frac{6}{7} - \frac{5}{21} \\ \frac{4}{21} - \frac{3}{7} + \frac{5}{21} & \frac{2}{7} + \frac{6}{7} - \frac{1}{7} & -\frac{26}{21} + \frac{9}{7} - \frac{1}{21} \\ -\frac{2}{21} - \frac{1}{7} + \frac{5}{21} & -\frac{1}{7} + \frac{2}{7} - \frac{1}{7} & +\frac{13}{21} + \frac{3}{7} - \frac{1}{21} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2-6+25}{21} & \frac{\cancel{?}+5-5}{7} & \frac{-13+18-5}{21} \\ \frac{4-9+5}{21} & \frac{2+6-1}{7} & \frac{-26+27-1}{21} \\ \frac{-2-3+5}{21} & \frac{-2+2}{7} & \frac{13+9-1}{21} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= I$$

Q. $A = \begin{bmatrix} 8 & 4 & 2 \\ 2 & 9 & 4 \\ 1 & 2 & 8 \end{bmatrix}$

$C_{11} = (-1)^2 \begin{vmatrix} 9 & 4 \\ 2 & 8 \end{vmatrix} = (+1)(72-8) = 64$

$C_{12} = (-1)^3 \begin{vmatrix} 2 & 4 \\ 1 & 8 \end{vmatrix} = (-1)(16-4) = -12$

$C_{13} = (-1)^4 \begin{vmatrix} 2 & 9 \\ 1 & 2 \end{vmatrix} = (+1)(4-9) = -5$

$C_{21} = (-1)^3 \begin{vmatrix} 4 & 2 \\ 2 & 8 \end{vmatrix} = (-1)(32-4) = -28$

$C_{22} = (-1)^4 \begin{vmatrix} 8 & 2 \\ 1 & 8 \end{vmatrix} = (+1)(64-2) = 62$

$C_{23} = (-1)^5 \begin{vmatrix} 8 & 4 \\ 1 & 2 \end{vmatrix} = (-1)(16-4) = -12$

$C_{31} = (-1)^4 \begin{vmatrix} 4 & 2 \\ 9 & 4 \end{vmatrix} = (+1)(16-18) = -2$

$C_{32} = (-1)^5 \begin{vmatrix} 8 & 2 \\ 2 & 4 \end{vmatrix} = (-1)(32-4) = -28$

$$C_{33} = (-1)^6 \begin{vmatrix} 8 & 4 \\ 2 & 9 \end{vmatrix} = (+1)(72-8) = 64$$

$$\text{Matrix of cofactors} = \begin{bmatrix} 64 & -12 & -5 \\ -28 & 62 & -12 \\ -2 & -28 & 64 \end{bmatrix}$$

$$\text{Adjoint} = \begin{bmatrix} 64 & -28 & -2 \\ -12 & 62 & -28 \\ -5 & -12 & 64 \end{bmatrix}$$

$$|A| = 8(64) - 4(12) + 2(-5)$$
$$= 512 - 48 - 10$$
$$= 512 - 58$$
$$= 454$$

$$A^{-1} = \frac{\text{adj } A}{|A|} = \frac{1}{454} \begin{bmatrix} 64 & -28 & -2 \\ -12 & 62 & -28 \\ -5 & -12 & 64 \end{bmatrix}$$

Q. Find A satisfying the matrix equation

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A = \begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}^{-1}$$

$PAQ = R$

$A = P^{-1} R Q^{-1}$

$$= \begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} -3 & -2 \\ -5 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -6+8 & -10+12 \\ 9-2 & 15-3 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 7 & 12 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 2 & 2 \\ 7 & 12 \end{bmatrix} \begin{bmatrix} -2 & 3 \\ 1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -4+2 & 6-4 \\ -14+12 & 21-24 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 2 \\ - & -3 \end{bmatrix}$$

$AB \neq BA$

$$\begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix}^{-1} = \begin{bmatrix} -1 & -4 \\ -3 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} -3 & -2 \\ -5 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 6-20 & +4-12 \\ -18+5 & -6+3 \end{bmatrix}$$

$$\begin{array}{r} 28 \\ 13 \\ \hline 15 \end{array}$$

$$\begin{bmatrix} 2a+c & 2b+d \\ 3a+2c & 3b+2d \end{bmatrix} = \begin{bmatrix} -14 & -8 \\ -13 & -3 \end{bmatrix}$$

$$4a+2c = -28$$
$$3a+2c = -13$$
$$- \quad + \quad +$$
$$a = -15$$

$$\begin{bmatrix} 24 & 13 \\ -34 & -18 \end{bmatrix}$$

Q. If $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 9 & 3 \\ 1 & 4 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & -1 \\ +1 & -1 & 3 \end{bmatrix}$

$$AB = \begin{bmatrix} 1+2+2 & 2+3-2 & 0-1+6 \\ 1+18+3 & 2+27-3 & 0-9+9 \\ 1+8+4 & 2+12-4 & 0-4+12 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 3 & 5 \\ 22 & 26 & 0 \\ 13 & 10 & 8 \end{bmatrix}$$

$$c_{11} = (-1)^2 \left[26(8) - 0 \right] = 208$$

$$c_{12} = (-1)^3 \left[22(8) - 0 \right] = -176$$

$$C_{13} = (-1)^4 [220 - 26(13)] = -118$$

$$C_{21} = (-1)^3 [24 - 50] = 26$$

$$C_{22} = (-1)^4 (40 - 65) = -25$$

$$C_{23} = (-1)^5 (50 - 39) = -11$$

$$C_{31} = (-1)^4 (0 - 26(5)) = -130$$

$$C_{32} = (-1)^5 (0 - 110) = 110$$

$$C_{33} = (-1)^6 [26(5) - 22(3)] = 64$$

~~Adjoint~~ Matrix of cofactors $= \begin{bmatrix} 208 & -176 & -118 \\ 26 & -25 & -11 \\ -130 & 110 & 64 \end{bmatrix}$

$$\text{Adjoint} = \begin{bmatrix} 208 & 26 & -130 \\ -176 & -25 & 110 \\ -118 & -11 & 64 \end{bmatrix}$$

$$\text{Det}(A) = 5(208) - 3(176) + 5(-118)$$
$$= 1040 - 528 - 590$$
$$= -78$$

$$A^{-1} = \begin{bmatrix} \frac{208}{-78} & 26/-78 & -130/-78 \\ \frac{-176}{-78} & -25/-78 & 110/-78 \\ -118/-78 & -11/-78 & 64/-78 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 9 & 3 \\ 1 & 4 & 4 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 9 & 3 \\ 1 & 4 & 4 \end{bmatrix}$$

$C_{11} = (-1)^2 (36-12) = 24$

$C_{12} = (-1)^3 (4-3) = -1$

$C_{13} = (-1)^4 (4-9) = -5$

$C_{21} = (-1)^3 (4-8) = 4$

$C_{22} = (-1)^4 (4-2) = 2$

$C_{23} = (-1)^5 (4-1) = -3$

$C_{31} = (-1)^4 (3-18) = -15$

$C_{32} = (-1)^5 (3-2) = -1$

$C_{33} = (-1)^6 (9-1) = 8$

Matrix of cofactors

$$= \begin{bmatrix} 24 & -1 & -5 \\ 4 & 2 & -3 \\ -15 & -1 & 8 \end{bmatrix}$$

~~|A|~~ Adjoint $= \begin{bmatrix} 24 & 4 & -15 \\ -1 & 2 & -1 \\ -5 & -3 & 8 \end{bmatrix}$

$|A| = 1(24) - \cancel{4}(1) + 2(\cancel{-5})$

$= 24 - 1 \cancel{-} \cancel{1}0$

$= \cancel{24} = 24 - 11 = 13$

$$A^{-1} = \frac{1}{13} \begin{bmatrix} 24 & 4 & -15 \\ -1 & 2 & -1 \\ -5 & -3 & 8 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

$C_{11} = (-1)^{2} \Big[$

9/9/15

Solution of simultaneous linear equations by matrix method.

Consider the following equations

$a_1x + a_2y + a_3z = d_1$
$b_1x + b_2y + b_3z = d_2$
$c_1x + c_2y + c_3z = d_3$

$AX = B$
$X = A^{-1}B$

where $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$; $B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$

Q. Solve the following equations

$x - y + z = 4$
$x - 2y - 2z = 9$
$2x + y - 3z = 1$

$A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & +3 \end{bmatrix}$ $B = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & +3 \end{bmatrix}^{-1} \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

Sol $\Delta = 1(-6+2) - \{-1[+3+4]\} + 1(1+4)$
$= -4 + 7 + 5$
$= 8$

$$C_{11} = (-1)^2 \begin{vmatrix} -2 & -2 \\ 1 & +3 \end{vmatrix} = -6+2 = -4$$

$$C_{12} = (-1)^3 \begin{vmatrix} 1 & -2 \\ 2 & +3 \end{vmatrix} = -1(3+4) = -7$$

$$C_{13} = (-1)^4 \begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} = 1+4 = 5$$

$$C_{21} = (-1)^3 \begin{vmatrix} -1 & 1 \\ 1 & 3 \end{vmatrix} = -1(-3-1) = 4$$

$$C_{22} = (-1)^4 \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = 3-2 = 1$$

$$C_{23} = (-1)^5 \begin{vmatrix} 1 & -1 \\ 2 & +1 \end{vmatrix} = -1 (1+2) = -3$$

~~Ac to Matrix of cofactor = |−4 −7 5~~

$$C_{31} = (-1)^4 \begin{vmatrix} -1 & 1 \\ -2 & -2 \end{vmatrix} = 2+2 = 4$$

$$C_{32} = (-1)^5 \begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} = -1(-2-1) = 3$$

$$C_{33} = (-1)^6 \begin{vmatrix} 1 & -1 \\ 1 & -2 \end{vmatrix} = -2+1 = -1$$

Matrix of cofactor $= \begin{bmatrix} -4 & -7 & 5 \\ 4 & 1 & -4 \\ 4 & 3 & -1 \end{bmatrix}$

Adjoint $= \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$

$$A^{-1} = \frac{adj\,A}{|A|} = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$$

$$X = A^{-1}B$$

$$= \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}_{3\times3}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}_{3\times1}$$

$$\begin{array}{r} 37 \\ 20 \\ \hline \end{array} \qquad = \frac{1}{8}\begin{bmatrix} -16+36+4 \\ -28+9+3 \\ 20-27-1 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix}$$

$$\begin{array}{r} 28 \\ 12 \\ \hline 16 \end{array} \qquad = \frac{1}{8}\begin{bmatrix} 34 \\ -16 \\ -17 \end{bmatrix} \qquad = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 34/24 \\ -16/8 \; -2 \\ -17/8 \end{bmatrix} = \begin{bmatrix} 17/4 \\ -2 \\ -17/8 \end{bmatrix}$$

$x = 3$

$y = -2$

$z = -1$

$x_1 = \frac{35}{18}$

$x_2 = \frac{29}{18}$

$x_3 = \frac{5}{18}$

Q. Solve the equations by matrix method

$$2x_1 + 3x_2 + x_3 = 9$$
$$x_1 + 2x_2 \quad 3x_3 = 6$$
$$3x_1 + x_2 + 2x_3 = 8$$

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix} \quad d = \begin{bmatrix} 9 \\ 6 \\ 8 \end{bmatrix}$$

$$|A| = 2(4-3) + 3(2-9) + 1(1-6)$$
$$= 2 - 3(-7) + 1(-5)$$
$$= 2 + 21 - 5$$
$$= 18$$

$$C_{11} = (-1)^2 \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = 4-3 = 1$$

$$C_{12} = (-1)^3 \begin{vmatrix} 1 & 3 \\ 3 & 2 \end{vmatrix} = -1(2-9) = +7$$

$$C_{13} = (-1)^4 \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = 1-6 = -5$$

$$C_{21} = (-1)^3 \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} = -1(6-1) = -5$$

$$C_{22} = (-1)^4 \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} = 4-3 = 1$$

$$C_{23} = (-1)^5 \begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix} = -1(2-9) = +7$$

$$C_{31} = (-1)^4 \begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} = 9-2 = 7$$

$$C_{32} = (-1)^5 \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = -1(6-1) = -5$$

$$C_{33} = (-1)^6 \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = 4-3 = 1$$

$$\text{Matrix of Cofactors} = \begin{bmatrix} 1 & 7 & -5 \\ -5 & 1 & 7 \\ 7 & -5 & 1 \end{bmatrix}$$

$$\text{adj}A = \begin{bmatrix} 1 & -5 & 7 \\ 7 & 1 & -5 \\ 5 & 7 & 1 \end{bmatrix}$$

$$X = A^{-1}B$$
$$= \frac{1}{18} \begin{bmatrix} 1 & -5 & 7 \\ 7 & 1 & -5 \\ 5 & 7 & 1 \end{bmatrix} \begin{bmatrix} 9 \\ 6 \\ 8 \end{bmatrix}$$

$$\cancel{A^{-1}B} = \frac{1}{18} \begin{bmatrix} 9-30+56 \\ 63+6-40 \\ 45+42+8 \end{bmatrix}$$

$$= \frac{1}{18} \begin{bmatrix} 35 \\ 29 \\ 95 \end{bmatrix}$$

$30 - 9 = 21$

$56 - 21 = 35$

$69 - 40 = 29$

solve the following equations by matrix method

$$2x - 2y + z = 2$$
$$3x + y - z = 0$$
$$x + 3y + 2z = 2$$

$$A = \begin{bmatrix} 2 & -2 & 1 \\ 3 & 1 & -1 \\ 1 & 3 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$|A| = 2(2+3) + 2(6+1) + 1(9-1)$$
$$= 2(5) + 14 + 8$$
$$= 24 + 8 = 32.$$

$$C_{11} = (-1)^2 \begin{vmatrix} 1 & -1 \\ 3 & 2 \end{vmatrix} = 2+3 = 5$$

$$C_{12} = (-1)^3 \begin{vmatrix} 3 & -1 \\ 1 & 2 \end{vmatrix} = -1(6+1) = -7$$

$$C_{13} = (-1)^4 \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} = 9-1 = 8$$

$$C_{21} = (-1)^3 \begin{vmatrix} -2 & 1 \\ 3 & 2 \end{vmatrix} = -1(-4-3) = 7$$

$$C_{22} = (-1)^4 \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 4-1 = 3$$

$$C_{23} = (-1)^5 \begin{vmatrix} 2 & -2 \\ 1 & 3 \end{vmatrix} = -1(6+2) = -8$$

$$C_{31} = (-1)^4 \begin{vmatrix} -2 & 1 \\ 1 & -1 \end{vmatrix} = 2 - 1 = 1$$

$$C_{32} = (-1)^5 \begin{vmatrix} 2 & 1 \\ 3 & -1 \end{vmatrix} = -1(-2-3) = 5$$

$$C_{33} = (-1)^6 \begin{vmatrix} 2 & -2 \\ 3 & 1 \end{vmatrix} = (2+6) = 8.$$

$$\text{Matrix of cofactors} = \begin{bmatrix} 5 & -7 & 8 \\ 7 & 3 & -8 \\ 1 & 5 & 8 \end{bmatrix}$$

$$\text{adj } A = \begin{bmatrix} 5 & 7 & 1 \\ -7 & 3 & 5 \\ 8 & -8 & 8 \end{bmatrix}$$

$$X = A^{-1} B$$

$$= \frac{1}{32} \begin{bmatrix} 5 & 7 & 1 \\ -7 & 3 & 5 \\ 8 & -8 & 8 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$$

$$= \frac{1}{32} \begin{bmatrix} 10 + 0 + 2 \\ -14 + 0 + 10 \\ 16 - 0 + 16 \end{bmatrix}$$

$$= \frac{1}{32} \begin{bmatrix} 12 \\ -4 \\ 32 \end{bmatrix} = \begin{bmatrix} 12/32 \\ -4/32 \\ 32/32 \end{bmatrix} = \begin{bmatrix} 3/8 \\ -1/8 \\ 1 \end{bmatrix}$$

$=$

Solution of simultaneous equations by Cramer's rule (Determinant rule)

Consider the following equation.

$$a_1 x + b_1 y_1 + c_1 = d_1$$
$$a_2 x + b_2 y_1 + c_2 = d_2$$
$$a_3 x + b_3 y_1 + c_3 = d_3$$

$$D = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \qquad D_1 = \begin{bmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{bmatrix}$$

$$D_2 = \begin{bmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{bmatrix} \qquad D_3 = \begin{bmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{bmatrix}$$

$$x = \frac{D_1}{D} \qquad y = \frac{D_2}{D} \qquad z = \frac{D_3}{D}$$

Q. $x - y + z = 4$
$x - 2y - 2z = 9$
$2x + y + 3z = 1$

$$D = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} = 1(-6+2) + 1(3+4) + 1(1+4)$$
$$= -4 + 7 + 5 = 8$$

$$D_1 = \begin{vmatrix} 4 & -1 & 1 \\ 9 & -2 & -2 \\ 1 & 1 & 3 \end{vmatrix} = 4(-6+2) + 1(27+2) + 1(9+2)$$

$$= -16 + 29 + 11$$

$$= -16 + 840$$

$$= 24$$

$$D_2 = \begin{vmatrix} 1 & 4 & 1 \\ 1 & 9 & -2 \\ 2 & 1 & 3 \end{vmatrix} = 1(27+2) - 4(3+4) + 1(1-18)$$

$$= 29 - 28 - 17$$

$$= -16$$

$$D_3 = \begin{vmatrix} 1 & -1 & 4 \\ 1 & -2 & 9 \\ 2 & 1 & 1 \end{vmatrix} = 1(-2-9) + 1(1-18) + 4(1+4)$$

$$= -11 - 17 + 20$$

$$= -28 + 20 = -8.$$

$$x = \frac{\cancel{24}^{\,3}}{\cancel{8}} \qquad y = \frac{-\cancel{16}^{\,2}}{\cancel{8}} \qquad z = \frac{-\cancel{8}^{\,1}}{\cancel{8}}$$

$$\boxed{x = 3} \qquad \boxed{y = -2} \qquad \boxed{z = -1}$$

(2). $5x - 6y + 4z = 15$

$7x + 4y - 3z = 19$

$2x + y + 6z = 46.$

Scanned by CamScanner

$D_2 = 1676$
$D_3 = 2514$

$x = 3$
$y = 4$
$z = 6.$

270
135
405

1814
19
165

$$D = \begin{vmatrix} 5 & -6 & 4 \\ 7 & 4 & -3 \\ 2 & 1 & 6 \end{vmatrix} = 5(24+3) + 6(42+6) + 4(7-8)$$

$$= 135 + 288 - 4$$
$$= 423 - 4$$
$$= 419.$$

$$D_1 = \begin{vmatrix} 15 & -6 & 4 \\ 19 & 4 & -3 \\ 46 & 1 & 6 \end{vmatrix} = 15(24+3) + 6[19(6) + 46(3)] + 4[19(1) - 4(46)]$$

1257

$$= 405 + 6[114 + 138] + 4[19 - 184]$$

$$= 405 + 6[252] - 4[165]$$

$$= 405 + 1512 - 660$$

$$= \cancel{2729} \quad \cancel{1577} \quad 1257$$

$$D_2 = \begin{vmatrix} 5 & 15 & 4 \\ 7 & 19 & -3 \\ 2 & 46 & 6 \end{vmatrix} = 5[19(6) + 46(3)] - 15(42+6) + 4[(46)(7) - 19(2)]$$

322
-38
284

$$= 5(114 + 138) - 15(48) + 4[322 - 38]$$

$$= 5[252] - 720 + 4[284]$$

$$= 1260 - 720 + 936.$$

$$= 540 + 936 = 1676$$

$$D_3 = \begin{vmatrix} 5 & -6 & 15 \\ 7 & 4 & 19 \\ 2 & 1 & 46 \end{vmatrix}$$

Q. 2 $x + y + z = 1$

$3x + 5y + 6z = 4$

$9x + 2y - 36z = 17$

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 5 & 6 \\ 9 & 2 & -36 \end{vmatrix} = 1\left[-36(5) - 12\right] - 1\left[-36(3) - 54\right] + 1(6 - 45)$$

$$= 1\left[-180 - 12\right] - 1\left[-108 - 54\right]$$

$$= -192 + 162 + 51$$

$$= -192 + 213$$

$$= 21$$

$$D_1 = \begin{vmatrix} 1 & 1 & 1 \\ 4 & 5 & 6 \\ 17 & 2 & -36 \end{vmatrix} = 1\left[-36(5) - 12\right] - 1\left[-36(4) - 17(6)\right] + 1(8 - 85)$$

$$\begin{array}{r} \cancel{246} \\ 177 \end{array} \qquad \begin{array}{r} \cancel{246} \\ 177 \\ \hline 69 \end{array}$$

$= (-180-12) - 1(-144-102) + 1(-77)$

$= (-192) - 1(-246) - 77$

$= -192 + 246 - 77$

$= -269 + 246 = -23.$

$$D_2 = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 4 & 6 \\ 9 & 17 & -36 \end{vmatrix} = 1[-36(4) - (17)(6)] - 1[-36(3) - 54] + 1(51 - 36)$$

$= [-144 - 102] - 1\{108 - 54) + 1(15)$

$= -246 + 1(162) + 15$

$= -246 + 177$

$= -69$

$$D_3 = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 5 & 4 \\ 9 & 2 & 17 \end{vmatrix} = 1[17(5) - 8] - 1[51 - 36] + 1[6 - 45]$$

$= [85 - 8] + 15 - 39$

$= 77 + 15 - 39.$

$= 92 - 39 = 23$

$D = -69$	$x = \frac{1}{3}$
$D_1 = -23$	$y = 1$
$D_2 = -69$	$z = -\frac{1}{3}$
$D_3 = 23$	

$x = \frac{-23}{-69} = \frac{1}{3}$ $\qquad y = \frac{-69}{-69} = 1$ $\qquad z = \frac{23}{-69} = -\frac{1}{3}$

15/9/15

Properties of a determinant

(1) Let $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

$\Delta' = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$ $\quad \Delta = \Delta'$

(2) $\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$ $\quad \Delta_1 = \begin{vmatrix} c_1 & c_2 & c_3 \\ b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \end{vmatrix}$

$\Delta_1 = -\Delta$

(3) $\Delta' = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$ two row/column are same

(4) $\Delta = \begin{vmatrix} ka_1 & kb_1 & kc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ $\quad \Delta' = k\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

(5) $\begin{vmatrix} a_1+p_1+q_1 & b_1 & c_1 \\ a_2+p_2+q_2 & b_2 & c_2 \\ a_3+p_3+q_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} p_1 & b_1 & c_1 \\ p_2 & b_2 & c_2 \\ p_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} q_1 & b_1 & c_1 \\ q_2 & b_2 & c_2 \\ q_3 & b_3 & c_3 \end{vmatrix}$

$-4+1+x$

Q. For what value of x is the matrix

$$\begin{bmatrix} 3-x & 2 & 2 \\ 1 & 4-x & 1 \\ -2 & -4 & -1-x \end{bmatrix} \text{ singular?}$$

$$\cancel{|A| = (3-x)\left[(4-x)(-1-x)+4\right]} - 2\left[\cancel{(-1-x)+2}\right]$$

$$\cancel{+2\left[-4+2(4-x)\right]}$$

$$\cancel{= 3-x \left\{ C_2 \to C_2 - C_3 \right.}$$

$$C_2 \to C_2 - C_3 \quad \begin{vmatrix} 3-x & 0 & 2 \\ 1 & 3-x & 1 \\ -2 & -3+x & -1-x \end{vmatrix}$$

$$= (3-x) \begin{vmatrix} 3-x & 0 & 2 \\ 1 & 1 & 1 \\ -2 & -1 & -1-x \end{vmatrix}$$

$R_2 \to R_2 + R_3$

$$= (3-x) \begin{vmatrix} 3-x & 0 & 2 \\ -1 & 0 & 1 \\ -2 & -1 & -1-x \end{vmatrix}$$

$\cancel{C_2 \to C_1 + C_3 - C_1}$ $\quad C_2 \to C_2 - C_3$

$C_3 \to C_3 - C_1$

$= (3-x)\left[+1\left(3-x+2\right)\right]$

$= \cancel{3x}\,(3-x)\,(5-x)$

$= \cancel{15}$

Applying $R_1 + R_2 + R_3$

$$= \begin{bmatrix} 2-x & 2-x & 2-x \\ 1 & 4-x & 1 \\ -2 & -4 & -1-x \end{bmatrix}$$

$$= (2-x)\begin{vmatrix} 1 & 1 & 1 \\ 1 & 4-x & 1 \\ -2 & -4 & -1-x \end{vmatrix}$$

$C_2 \to C_2 - C_3 \qquad C_3 \to C_3 - C_1$

$$= 2-x\begin{vmatrix} 1 & 0 & 0 \\ 1 & 3-x & 0 \\ -2 & -3+x & 1-x \end{vmatrix}$$

$= (2-x)\left[1\left((3-x)(1-x)\right) - 0\right]$

$\Rightarrow (2-x)\left[(3-x)(1-x)\right] = 0$

$x = 1, 2, 3$

Prove that

$$\begin{vmatrix} x & a & a \\ a & x & a \\ a & a & x \end{vmatrix} = (x+2a)(x-a)^2.$$

$$C_1 \to C_1 - C_2$$

$$= \begin{vmatrix} x-a & a & a \\ a-x & x & a \\ 0 & a & x \end{vmatrix}.$$

$$= (x-a) \begin{vmatrix} 1 & a & a \\ -1 & x & a \\ 0 & a & x \end{vmatrix}$$

$$C_2 \to C_2 - C_3$$

$$= (x-a)^2 \begin{vmatrix} 1 & 0 & a \\ -1 & x-a & a \\ 0 & a-x & x \end{vmatrix}$$

$$= (x-a)^2 \begin{vmatrix} 1 & 0 & a \\ -1 & 1 & a \\ 0 & -1 & x \end{vmatrix}$$

$\cancel{R_1 \to R_1 + R_2}$

$= (x-a)^2$ | (

IInd method

$R_1 \to R_1 + R_2 + R_3$

$$= \begin{vmatrix} x+2a & x+2a & x+2a \\ a & x & a \\ a & a & x \end{vmatrix}$$

$$= (x+2a) \begin{vmatrix} 1 & 1 & 1 \\ a & x & a \\ a & a & x \end{vmatrix}$$

$C_1 \to C_1 - C_2$

$$= (x+2a) \begin{vmatrix} 0 & 1 & 1 \\ a-x & x & a \\ 0 & a & x \end{vmatrix}$$

$C_2 \to C_2 - C_3$

$$= (x+2a) \begin{vmatrix} 0 & 0 & 1 \\ a-x & x-a & a \\ 0 & a-x & x \end{vmatrix}$$

$= \cancel{(x+2a)(a-x)}$

$$= (x+2a)(x-a)^2 \begin{vmatrix} 0 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & -1 & x \end{vmatrix} = (x+2a)(x-a)$$

$= (a-b)(b-c)\left[-(a-c)\{1\}\right]$

$= (a-b)(b-c)(c-a)$

(ii)
$$\begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix}$$

$$= abc \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$R_1 \longrightarrow R_1 - R_2, \; R_2 \longrightarrow R_2 - 3$

$$= abc \begin{vmatrix} 0 & a-b & a^2-b^2 \\ 0 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$= (abc)(a-b)(b-c) \begin{vmatrix} 0 & 1 & a+b \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$$

$R_1 \longrightarrow R_1 - R_2$

$$= (abc)(a-b)(b-c) \begin{vmatrix} 0 & 0 & a-c \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$$

$= (abc)(a-b)(b-c)\left[-(a-c)\right.$

$= (abc)(a-b)(b-c)(c-a)$

I + II

$(a-b)(b-c)(c-a) + (abc)(a-b)(b-c)(c-a)$

$= (1+abc)(a-b)(b-c)(c-a).$

= Hence Proved.

IInd method

$$\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix}$$

$$= \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix}$$

$$= \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + abc \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$C_1 \leftrightarrow C_2, \quad C_2 \leftrightarrow C_3.$

$$= (1+abc) \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}$$

~~= (1+abc~~

~~R~~ $R_1 - R_2 + R_3$

$R_1 \to R_1 - R_2 \quad , \quad R_2 \to R_2 - R_3$

$$= (1+abc)\begin{vmatrix} a-b & a^2-b^2 & 0 \\ b-c & b^2-c^2 & 0 \\ c & c^2 & 1 \end{vmatrix}$$

$$= (1+abc)(a-b)(b-c)\begin{vmatrix} 1 & a+b & 0 \\ 1 & b+c & 0 \\ c & c^2 & 1 \end{vmatrix}.$$

$R_1 \longrightarrow R_1 - R_2$

$$= (1+abc)(a-b)(b-c)\begin{vmatrix} 0 & a-c & 0 \\ 1 & b+c & 0 \\ c & c^2 & 1 \end{vmatrix}$$

$$= (1+abc)(a-b)(b-c)\left[-(a-c)\right]$$

$$= (1+abc)(a-b)(b-c)(c-a)$$ Hence Prove

Q.1 Solve the equation $\begin{vmatrix} x^3-a^3 & x^2 & x \\ b^3-a^3 & b^2 & b \\ c^3-a^3 & c^2 & c \end{vmatrix} = 0.$

$b \neq c$
$bc \neq 0.$

Q.2. Solve the eq^n. $\begin{vmatrix} 2x-1 & x+7 & x+4 \\ x & 6 & 2 \\ x-1 & x+1 & 3 \end{vmatrix} = 0$

$\boxed{x = b}$ $\boxed{x = c}$ $xbc = a^3$

$$\boxed{x = \frac{a^3}{bc}}$$

(2)
$$\begin{vmatrix} 2x-1 & x+7 & x+4 \\ x & 6 & 2 \\ x-1 & x+1 & 3 \end{vmatrix} = 0$$

$R_1 \to R_1 - R_2 + R_3$

$$= \begin{vmatrix} 2x-1-\cancel{x}+\cancel{x}-1 & x+7-6+x+1 & x+4-2+3 \\ x & 6 & 2 \\ x-1 & x+1 & 3 \end{vmatrix}$$

$$= \begin{vmatrix} 2x-2 & 2x+2 & x+5 \\ x & 6 & 2 \\ x-1 & x+1 & 3 \end{vmatrix}$$

$$x = \pm\sqrt{a^2+b^2+c^2-ab-bc-ca}$$

5) Show that $x = -(a+b+c)$ is one root of the equation and solve the eq^n completely

$$\begin{vmatrix} x+a & b & c \\ b & x+c & a \\ c & a & x+b \end{vmatrix} = 0$$

$R_1 \to R_1 + R_2 + R_3$

$$\begin{vmatrix} x+a+b+c & x+a+b+c & x+a+b+c \\ b & x+c & a \\ c & a & x+b \end{vmatrix} = 0$$

$$(x+a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ b & x+c & a \\ c & a & x+b \end{vmatrix} = 0.$$

$x+a+b+c=0$

$\boxed{x = -(a+b+c)}$

$C_1 \to C_1 - C_2 \quad , \quad C_2 \to C_2 - C_3$

$$\begin{vmatrix} 0 & 0 & 1 \\ b-x-c & x+c-a & a \\ c-a & a-x-b & x+b \end{vmatrix} = 0$$

$$1\left[(b-x-c)(a-x-b) - (x+c-a)(c-a)\right] = 0$$

$$\left[ba - bx - b^2 - xa + x^2 + xb - ca + cx + cb\right] - \left[xc - xa + c^2 - ca - ac + a^2\right] = 0$$

$x_1^2 = x_2^2$ $y \neq x_1$

$x_1 = x_2$

$ba - bx - b^2 - \cancel{x\alpha} + x^2 + xb - c\alpha + \cancel{c\alpha} + cb$
$- \cancel{c\alpha} + \cancel{x}a - c^2 + ca + \cancel{c}a + a^2 = 0$

$ba - bx - b^2 + x^2 + xb + bc - c^2 + ac + a^2 = 0$

$x^2 - a^2 - b^2 - c^2 + bc + ac + ab = 0$

$x = \pm\sqrt{a^2 + b^2 + c^2 - ab - bc - ca}$

Q. Evaluate $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$.

$R_1 \to R_1 + R_2 + R_3$

$= \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

$= (a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

$C_1 \to C_1 - C_2$ $C_2 \to C_2 - C_3$

$= (a+b+c) \begin{vmatrix} 0 & 0 & 1 \\ b+c+a & -a-b-c & 2b \\ 0 & a+b+c & c-a-b \end{vmatrix}$

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$$=(a+b+c)^3 \begin{vmatrix} 0 & 0 & 1 \\ 1 & -1 & 2b \\ 0 & 1 & c-a-b \end{vmatrix}.$$

$$=(a+b+c)^3$$

Q. Prove that.

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+a & 1 \\ 1 & 1 & 1+a \end{vmatrix} = a^3+3a^2.$$

$C_1 \to C_1 - C_2 \quad C_2 \to C_2 - C_3$

$$= \begin{vmatrix} a & 0 & 1 \\ -a & a & 1 \\ 0 & -a & 1+a \end{vmatrix}$$

$$= a^2 \begin{vmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & -1 & 1+a \end{vmatrix}.$$

$C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} 3+a & 1 & 1 \\ 3+a & 1+a & 1 \\ 3+a & 1 & 1+a \end{vmatrix}$$

$$= (3+a)\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+a & 1 \\ 1 & 1 & 1+a \end{vmatrix}$$

$C_1 \to C_1 - C_2 \qquad C_2 \to C_2 - C_3$

$$= (3+a)\begin{vmatrix} 0 & 0 & 1 \\ -a & a & 1 \\ 0 & -a & 1+a \end{vmatrix}$$

$$= (3+a)(a^2\begin{vmatrix} 0 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & -1 & 1+a \end{vmatrix}$$

$$= (3+a)(a^2)\left[1-0\right)$$

$$= a^3 + 3a^2.$$

Q. Prove that $\begin{vmatrix} 1+a_1 & 1 & 1 \\ 1 & 1+a_2 & 1 \\ 1 & 1 & 1+a_3 \end{vmatrix}$

$= a_1a_2a_3\left(1+\frac{1}{a_1}+\frac{1}{a_2}\right.$

$$= a_1a_2a_3\begin{vmatrix} 1/a_1+1 & 1/a_2 & 1/a_3 \\ 1/a_1 & 1/a_2+1 & 1/a_3 \\ 1/a_1 & 1/a_2 & 1/a_3+1 \end{vmatrix}$$

$C_1 - C_3$
$C_2 - C_3$

$C_1 \rightarrow C_1 + C_2 + C_3$

$$= a_1 a_2 a_3 \begin{vmatrix} 1/a_1 + 1 + 1/a_2 + 1/a_3 & 1/a_2 & 1/a_3 \\ 1/a_1 + 1 + 1/a_2 + 1/a_3 & 1/a_2 + 1 & 1/a_3 \\ 1/a_1 + 1 + 1/a_2 + 1/a_3 & 1/a_2 & \frac{1}{a_3} + 1 \end{vmatrix}$$

$$= a_1 a_2 a_3 \left(1 + \frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3}\right) \begin{vmatrix} 1 & 1/a_2 & 1/a_3 \\ 1 & 1/a_2 + 1 & 1/a_3 \\ 1 & 1/a_2 & 1/a_3 + 1 \end{vmatrix}$$

$R_1 \rightarrow R_1 - R_2$
$R_2 \rightarrow R_2 - R_3$

$$= a_1 a_2 a_3 \left(1 + \frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3}\right) \begin{vmatrix} 0 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 1/a_2 & 1/a_3 + 1 \end{vmatrix}$$

$$= a_1 a_2 a_3 \left(1 + \frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3}\right) [1 - 0]$$

$$= a_1 a_2 a_3 \left(1 + \frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3}\right)$$

$c - a + b + c + a - b$
$2c - 2c$

Q. $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b) \end{vmatrix}$

$c-a-b-c+a$

$-2b$

$R_3 \to R_3 - R_1 + R_2$

$c-a-b-b-c+a$

$-2b$

$C_1 \to C_1 - C_3 \;,\; C_2 \to C_2 - C_3$

$$= \begin{vmatrix} (b+c)^2 - a^2 & 0 & a^2 \\ 0 & [(c+a)^2 - b^2) & b^2 \\ c^2 - (a+b)^2 & c^2 - (a+b)^2 & (a+b)^2 \end{vmatrix}$$

$$= \begin{vmatrix} (b+c-a)(b+c+a) & 0 & a^2 \\ 0 & (c+a-b)(c+a+b) & b^2 \\ (c-a-b)(c+a+b) & (c-a-b)(c+a+b) & (a+b)^2 \end{vmatrix}$$

$$= (a+b+c)^2 \begin{vmatrix} b+c-a & 0 & a^2 \\ 0 & c+a-b & b^2 \\ c-a-b & c-a-b & (a+b)^2 \end{vmatrix}$$

$R_3 \to R_3 - (R_1 + R_2)$

$$= (a+b+c)^2 \begin{vmatrix} b+c-a & 0 & a^2 \\ 0 & c+a-b & b^2 \\ -2b & 2c-2b & (a+b)^2 - a^2 + b^2 \end{vmatrix}$$

Q. Solve the equation by Cramer's Rule

$$x+y+z=1$$
$$ax+by+cz=K$$
$$a^2x+b^2y+c^2z=K^2.$$

provided that $a\neq b,\ b\neq c$

$$D=\begin{vmatrix}1 & 1 & 1\\ a & b & c\\ a^2 & b^2 & c^2\end{vmatrix}$$

$C_1 \to C_1 - C_2 \quad C_2 \to C_2 - C_3$

$$= \begin{vmatrix}0 & 0 & 1\\ a-b & b-c & c\\ a^2-b^2 & b^2-c^2 & c^2\end{vmatrix}$$

$$(a-b)(b-c)\begin{vmatrix}0 & 0 & 1\\ 1 & 1 & c\\ a+b & b+c & c^2\end{vmatrix}$$

$$\cancel{(a+b)}\ (a-b)(b-c)\left(\cancel{b}+c-a-\cancel{b}\right)$$

$$\boxed{D = (a-b)(b-c)(c-a)}$$

$$D_1=\begin{vmatrix}1 & 1 & 1\\ K & b & c\\ K^2 & b^2 & c^2\end{vmatrix}$$

$C_1 \to C_1 - C_2$, $C_2 \to C_2 - C_3$

$$= \begin{vmatrix} 0 & 0 & 1 \\ k-b & b-c & c \\ k^2-b^2 & b^2-c^2 & c^2 \end{vmatrix}$$

$$= (k-b)(b-c) \begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & c \\ k+b & b+c & c^2 \end{vmatrix}$$

$$= (k-b)(b-c)(\cancel{b}+c-k-\cancel{b})$$

$$\boxed{D_1 = (k-b)(b-c)(c-k)}$$

$$D_2 = \begin{vmatrix} 1 & 1 & 1 \\ a & k & c \\ a^2 & k^2 & c^2 \end{vmatrix}$$

$C_1 \to C_1 - C_2$ C_2.

$$= \begin{vmatrix} 0 & 0 & 1 \\ a-k & k-c & c \\ a^2-k^2 & k^2-c^2 & c^2 \end{vmatrix}$$

$$= (a-k)(k-c) \begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & c \\ a+k & k+c & c^2 \end{vmatrix}$$

$$= (a-k)(k-c)(\cancel{k}+c-a-\cancel{k})$$

$$\boxed{D_2 = (a-k)(k-c)(c-a)}$$

$$D_3 = \begin{vmatrix} 1 & 1 & 1 \\ a & b & k \\ a^2 & b^2 & k^2 \end{vmatrix}$$

$C_1 \longrightarrow C_1 - C_2 \,,\; C_2 \rightarrow C_2 - C_3$

$$= \begin{vmatrix} 0 & 0 & 1 \\ a-b & b-k & k \\ a^2-b^2 & b^2-k^2 & k^2 \end{vmatrix}$$

$$= (a-b)(b-k) \begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & k \\ a+b & b+k & k^2 \end{vmatrix}$$

$$= (a-b)(b-k)\left(\cancel{b}+k-a-\cancel{b}\right)$$

$$= (a-b)(b-k)(k-a)$$

$$x = \frac{D_1}{D} = \frac{(k-b)\cancel{(b-c)}(c-k)}{(a-b)\cancel{(b-c)}(c-a)}$$

$$\boxed{x = \frac{(k-b)(c-k)}{(a-b)(c-a)}}$$

$$y = \frac{D_2}{D} = \frac{(a-k)(k-c)\cancel{(c-a)}}{(a-b)(b-c)\cancel{(c-a)}}$$

$$\boxed{y = \frac{(a-k)(k-c)}{(a-b)(b-c)}}$$

$$z = \frac{D_3}{D} = \frac{\cancel{(a-b)}(b-k)(k-a)}{\cancel{(a-b)}(b-c)(c-a)}$$

$$\boxed{z = \frac{(b-k)(k-a)}{(b-c)(c-a)}}$$

Binomial Theorem

$$(a+b)^n = \sum_{r=0}^{n} {}^nC_r \, a^{n-r} b^r$$

General Term, $T_{(r+1)} = {}^nC_r \, a^{n-r} b^r$.

For any any positive integral index n

$$(x+y)^n = {}^nC_0 x^n + {}^nC_1 x^{n-1} y + {}^nC_2 x^{n-2} y^2 + {}^nC_3 x^{n-3} y^3 + \dots {}^nC_r x^{n-r} y^r + \dots {}^nC_n y^n$$

${}^nC_r x^{n-r} y^r$ is called general term

$$\boxed{T_{r+1} = {}^nC_r x^{n-r} y^r}$$

$\frac{6!}{0!(6-0)!} = \frac{6!}{6!} = 1$ $\frac{6!}{1!5!}$

Expand.

$^nC_r = \frac{n!}{r!(n-r)!}$

$\left(\frac{2x}{3} - \frac{3}{2x}\right)^6$

$0! = 1$

$1! = 1$

$0! = 1!$

$= {}^6C_0\left(\frac{2x}{3}\right)^6 + {}^6C_1\left(\frac{2x}{3}\right)^5\left(\frac{-3}{2x}\right)^1$

$+ {}^6C_2\left(\frac{2x}{3}\right)^4\left(\frac{-3}{2x}\right)^2 + {}^6C_3\left(\frac{2x}{3}\right)^3\left(\frac{-3}{2x}\right)^3 + {}^6C_4\left(\frac{2x}{3}\right)^2\left(\frac{-3}{2x}\right)^4$

$+ {}^6C_5\left(\frac{2x}{3}\right)^1\left(\frac{-3}{2x}\right)^5 + {}^6C_6\left(\frac{-3}{2x}\right)^6$

$= \frac{64x^6}{729} + 6\left(\frac{32x^5}{243}\right)\left(\frac{-3}{2x}\right)^1 + 15\left(\frac{16x^4}{81}\right)\left(\frac{9}{4x^2}\right)$

$+ 20\left(\frac{8x^3}{27}\right)\left(\frac{-27}{8x^3}\right) + 15\left(\frac{4x^2}{9}\right)\left(\frac{81}{16x^4}\right)$

$+ 6\left(\frac{2x}{3}\right)\left(\frac{-243}{32x^5}\right) + \left(\frac{729}{64x^6}\right)$

$= \frac{64x^6}{729} - \frac{32x^4}{27} + \frac{20}{3x^2} - 20 + \frac{135}{x^2}$

$- \frac{243}{8} + \frac{729}{64x^6}$

30/9/15

Q. $(2x+3y)^5$

$$= {}^5C_0(2x)^5 + {}^5C_1(2x)^4(3y)^1 + {}^5C_2(2x)^3(3y)^2$$
$$+ {}^5C_3(2x)^2(3y)^3 + {}^5C_4(2x)(3y)^4 + {}^5C_5(3y)^5$$

$$= 1(32x^5) + 5(16x^4)(3y) + 10(8x^3)(9y^2)$$
$$+ 10(4x^2)(27y^3) + 5(2x)(81y^4) + 243y^5$$

$$= 32x^5 + 240x^4y + 720x^3y^2 + 1080x^2y^3$$
$$+ 810xy^4 + 243y^5$$

Q. Find the 5^{th} term in expansion of $\left(\frac{4x}{5} - \frac{5}{2x}\right)^8$

$$T_{r+1} = {}^nC_r\, x^{n-r}\, y^r$$

$$T_{4+1} = {}^8C_4 \left(\frac{4x}{5}\right)^4 \left(\frac{-5}{2x}\right)^4$$

$$= \frac{\cancel{8}^{2}\times 7\times \cancel{6}\times 5\times \cancel{4!}}{\cancel{4}\,\cancel{3}\,\cancel{4!}\times \cancel{3\times 2}} \left(\frac{\cancel{256x^4}^{16}}{\cancel{625}}\right)\left(\frac{+\cancel{625}}{\cancel{16x^4}}\right)$$

$$= 70(16)$$

$$= 1120$$

Q. Find the 7^{th} term in the expansion of $\left(\frac{x^3}{2} - \frac{2}{x^2}\right)^9$

$$T_{6+1} = {}^9C_6 \left(\frac{x^3}{2}\right)^3 \left(-\frac{2}{x^2}\right)^6$$

$$= \frac{\overset{3\times 8}{\cancel{9}\times 7\times \cancel{6!}}}{\cancel{6!}\,\cancel{3}\times\cancel{2}} \left(\frac{x^{\cancel{9}}}{\cancel{8}}\right)\left(\frac{+\overset{4}{\cancel{64}}}{\cancel{x^{12}}\,x^3}\right)$$

$$= \frac{7\times 3\times 4\times 8}{x^3}$$

$$= \frac{672}{x^3}$$

08/10

Q. Find the 4^{th} term from the end in the expansion of $\left(\frac{x^3}{2} + \frac{2}{x^2}\right)^9$

t_7?

$$t_{r+1} = {}^9C_r \left(\frac{x^3}{2}\right)^{9-r} \left(\frac{2}{x^2}\right)^r$$

$$T_{6+1} = {}^9C_6 \left(\frac{x^3}{2}\right)^{9-6} \left(\frac{2}{x^2}\right)^6$$

$$= \frac{\overset{3}{\cancel{9}} \times \overset{4}{\cancel{8}} \times 7 \times \cancel{6!}}{\cancel{6!} \times \cancel{3} \times \cancel{2}} \left(\frac{x^9}{\cancel{8}}\right)\left(\frac{\overset{8}{\cancel{64}}}{x^{\cancel{12}\,3}}\right)$$

$$= 84\left(\frac{8}{x^3}\right)$$

$$= \frac{672}{x^3}$$

Q. Find the 4^{th} term from the end in $\left(\frac{4x}{5} - \frac{5}{2x}\right)^9$

T_7 ?

1 2 3 4 5 6 7 8 9 10

$$T_{6+1} = {}^9C_6 \left(\frac{4x}{5}\right)^3 \left(\frac{-5}{2x}\right)^6$$

$$= 84\left(\frac{\cancel{64}\, x^3}{\cancel{125}}\right)\left(\frac{\cancel{-125} \times 125}{\cancel{64}\, x^6}\right)$$

$$= 10500\, x^{-3}$$

$$= \frac{10500}{x^{-3}}$$

Middle Term

If n is even, middle term $= \left(\frac{n}{2}+1\right)^{th}$

n is odd, middle terms $= \left(\frac{n+1}{2}\right)^{th}$

& $\left(\frac{n+1}{2}+1\right)^{th}$

14400

Q. Find the middle term in the expansion of $\left(2x^2 - \frac{1}{3x^2}\right)^{10}$.

even, $\left(\frac{10}{2}+1\right)^{th} = 6^{th}$ term.

$T_6 = T_{5+1} = {}^{10}C_5 \left(2x^2\right)^5 \left(\frac{-1}{3x^2}\right)^5$.

$= \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5!}{5!\, 5 \times 4 \times 3 \times 2} \left(32x^{10}\right)\left(\frac{-1}{243x^{10}}\right)$

$\cancel{= 126 (32)\left(\frac{-1}{243}\right)}$

$\cancel{\frac{84}{27}}$

$\cancel{= -\frac{128}{27}}$

$= 252 \times \left(32x^{10}\right)\left(\frac{-1}{243x^{10}}\right)$

$= -\frac{896}{27}$

2. Find the middle term in the expansion of $\left(2x + \frac{3}{x}\right)^{20}$

Middle term

even, $\left(\frac{20}{2}+1\right)^{th} = 11^{th}$ term

$$T_{11} = T_{10+1} = {}^{20}C_{10}\,(2x)\left(\frac{\ }{x}\right)$$

$$= \frac{\overset{2}{\cancel{20}} \times \cancel{19} \times \cancel{18} \times 17 \times \cancel{16} \times \cancel{15} \times \cancel{14} \times 13 \times \cancel{12} \times 11 \times \cancel{10}}{\cancel{10!}\ \cancel{10!}\ \cancel{10} \times \cancel{9} \times \cancel{8} \times \cancel{7} \times \cancel{6} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2}} \left(1024\,\cancel{x^{10}}\right)\left(\frac{59049}{\cancel{x^{10}}}\right)$$

$$= 184756 \times 1024 \times 59049$$

$$= 1.117 \times 10^{13}$$

Q. Find the middle terms in the expansion of $\left(3x - \frac{x^3}{6}\right)^9$

Middle term $= \frac{9+1}{2} = 5$

$= 5 + 1 = 6.$

Middle terms are 5 and 6th te.

$$T_5 = t_{4+1} = {}^{9}C_4\,(3x)^5\left(-\frac{x^3}{6}\right)^4$$

$$= \frac{9 \times \cancel{8} \times 7 \times \cancel{6} \times \cancel{5} \times \cancel{4!}}{\cancel{4!} \times \cancel{5} \times \cancel{4} \times \cancel{3 \times 2}}\left(\overset{3}{\cancel{243}}\,x^5\right)\left(\frac{x^{12}}{\cancel{1296}\ 168}\right)$$

$$T_6 = T_{5+1} = {}^9C_5 (3x)^4 \left(\frac{-x^3}{6}\right)^5$$

$$= 126 \times 81x^4 \left(\frac{-x^{15}}{7776}\right)$$

$$= \frac{-21}{16} x^{19}$$

Q. Find the middle term in the expansion of $\left(x - \frac{1}{x}\right)^{11}$

$\left(x - \frac{1}{x}\right)^{11}$ Middle term $\frac{11+1}{2} = 6$

$6 + 1 = 7.$

$$T_6 = T_{5+1} = {}^{11}C_5 (x)^6 \left(\frac{-1}{x}\right)^5$$

$$= \frac{11 \times \cancel{10} \times \cancel{9} \times \cancel{8} \times \cancel{7} \times \cancel{6!}}{\cancel{5} \times \cancel{4} \times \cancel{6!} \times \cancel{3} \times \cancel{2}} (x^6)\left(\frac{-1}{\cancel{x^5}}\right)$$

$$= -462x.$$

$$T_7 = T_{6+1} = {}^{11}C_6 (x)^5 \left(-\frac{1}{x}\right)^6$$

$$= \frac{11 \times \cancel{10} \times \cancel{9} \times \cancel{8} \times 7 \times \cancel{6!}}{\cancel{6!}\ \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2}} (\cancel{x^5})\left(\frac{1}{x^{\cancel{6}}}\right)$$

$$= \frac{462}{x}$$

13/10

Q. Find

ex

13/10

Q. Find the term independent of x in the expansion of $\left(x^2 - \frac{2}{x}\right)^9$.

$$T_{r+1} = {}^nC_r (x)^{n-r} (y)^r$$

$$T_{r+1} = {}^9C_r (x^2)^{9-r} \left(\frac{-2}{x}\right)^r$$

$$= {}^9C_r (x)^{18-2r} \left(\frac{-2^r}{x^r}\right)$$

$$= {}^9C_r (x)^{18-2r} \left(\frac{-2}{x}\right)^r$$

$$= {}^9C_r (x)^{18-2r} (x)^{-r} (-2)^r$$

$$= {}^9C_r \left(x^{18-3r}\right) (-2)^r$$

$18 - 3r = 0$

$3r = 18$

$\boxed{r = 6}$

$$T_{6+1} = {}^9C_6 (x^2)^3 \left(\frac{-2}{x}\right)^6$$

$$= 84 \cancel{(x)^6} \left(\frac{2^6}{\cancel{x^6}}\right)$$

$$= 84 \times 64$$

$$= 5376$$

Q. Find the constant term in the expansion in the expansion of $\left(3x^2 - \frac{2}{x^3}\right)^{10}$

$$T_{r+1} = {}^{10}C_r \, (3x^2)^{10-r} \left(\frac{-2}{x^3}\right)^{r}$$

$$\cancel{= {}^{10}C_r \, (3x^2)^{-20-r} \, (-2)^{10}}$$

$$\cancel{-20-r =}$$

$$= {}^{10}C_r \, (3)^{10-r} \, x^{20-2r} \, (-2)^{r} \left(x^{-3r}\right)$$

$$\cancel{= {}^{10}C_r \, (3)^{10-r} \, x^{-10-2r} \, (-2)^{3r}}$$

$$\cancel{-10-2r=0} \quad {}^{10}C_r \, (3)^{10-r}$$

$$\cancel{2r = 10}$$

$$= {}^{10}C_r \, (3)^{10-r} \, x^{20-5r} \, (-2)^{3r}$$

$$20 - 5r = 0$$

$$5r = 20$$

$$\boxed{r = 4}$$

$$T_5 = {}^{10}C_4 \, (3x^2)^6 \left(\frac{-2}{x^3}\right)^4$$

$$= \frac{10 \times \cancel{9}^{3} \times \cancel{8}^{2} \times \cancel{7} \times \cancel{6} \times \cancel{5} \times \cancel{4!}}{\cancel{4!} \cdot \cancel{6} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2}} \left(729 \cancel{x^{12}}\right) \left(\frac{16}{\cancel{x^{12}}}\right)$$

$$= 2449440.$$

Q. Find the constant term in the expansion in the expression of $\left(3x^2 - \frac{1}{x^3}\right)^{20}$

$$T_{r+1} = {}^{20}C_r \left(3x^2\right)^{20-r} \left(\frac{-1}{x^3}\right)^{r}$$

$$= {}^{20}C_r \,(3)^{20-r}\, x^{40-2r}\, (-1)^r\, (x)^{-3r}$$

$$= {}^{20}C_r \,(3)^{20-r}\, (-1)^r\, x^{40-5r}$$

$$40 - 5r = 0$$

$$5r = 40$$

$$\boxed{r = 8}$$

$$T_9 = {}^{20}C_8 \left(3x^2\right)^{12} \left(\frac{-1}{x^3}\right)^{8}$$

$$= 125970 \times 531441 \times \cancel{x^{24}} \times \frac{1}{\cancel{x^{24}}}$$

$$= 6.69456 \times 10^{10}$$

14/10

Find the coefficient of x^{32} in the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$. Also find the coefficient of x^{-17}.

$T_{r+1} = {}^{15}C_r (x^4)^{15-r} \left(\frac{-1}{x^3}\right)^r$

$= {}^{15}C_r \; x^{60-4r} (-1)^r (x^{-3r}$.

$60 - 7r = 32$

$7r = 60 - 32$

$7r = 28$

$\boxed{r = 4}$

~~$T_{6+1} = {}^{15}C_6 (x^4)^9 \left(\frac{-1}{x^3}\right)^6$~~ coeff of $x^{32} =$

$T_{4+1} = \cancel{{}^{15}C_4 (x^4)^{11} \left(\frac{-1}{x^3}\right)^4}$ ${}^{15}C_4 (-1)^4 x^{32}$

$= \cancel{455} \; \cancel{1365} = 1365$

$60 - 7r = -17$

$7r = 60 + 17$

$7r = 77$

$\boxed{r = 11}$

Coeffecint of $x^{-17} = {}^{15}C_{11} (-1)^{11}$

$= -1365$

Q. Find the coefficient of x^{18} in the expansion of $\left(x^2+\frac{3a}{x}\right)^{15}$

$$T_{r+1} = {}^{15}C_r \,(x^2)^{15-r} \left(\frac{3a}{x}\right)^{r}$$

$$= {}^{15}C_r \,(x)^{30-2r} \,(3a)^{r}\, x^{-r}$$

$3\cancel{0-15-2r=18}$ $\quad$ $30-3r=18$

$\cancel{15-2r=18}$ $\quad$ $3r=30-18$

$\cancel{2r=}$ $\quad$ $3r=12$

$\boxed{r=4}$

Coefficient of $x^{18} = {}^{15}C_4 \,(3a)^4$

$= 1365 \times 81a^4$

$= 110565\, a^4.$

Q. Find the coefficient of x^{-3} in the expansion of $\left(\frac{4x}{5}-\frac{5}{2x}\right)^7$

$$T_{r+1} = {}^{7}C_r \left(\frac{4x}{5}\right)^{7-r} \left(\frac{-5}{2x}\right)^{r}$$

$$= {}^{7}C_r \,(4x)^{7-r}\,(5)^{r-7}\,(-5)^{r}\,(x^{-r})$$

$7-r-r=-3$

$7-2r=-3$

$2r=10$ $\quad$ $\boxed{r=5}$

Coefficient of $x^{-3} = \cancel{{}^7C_5 (4)(-5)}$

$= \cancel{21 \times 16 \times -3125}$

$= \cancel{{}^7C_5 \left(\frac{4x}{5}\right)^2 \left(\frac{-5}{x}\right)^5}$

$= {}^7C_5 \left(\frac{4}{5}\right)^2 \frac{(-5)^5}{2^5}$

$= \frac{7 \times 6 \times 5!}{5! \times 2} \left(\frac{16}{25}\right) \left(\frac{-3125}{32}\right)$

$= \frac{-2625}{2}$

125
21
125
250
2625

Binomial Theorem

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots$$

$$\dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r$$

General Term

$$T_{r+1} = \frac{n(n-1)(n-2)\dots(n-r+1)}{r!}x^r$$

Q. In the expansion of $\frac{1}{\sqrt{4-3x^2}}$

Find (i) the General Term

(ii) the 6^{th} term and

(iii) the coefficient of x^6.

$$(4-3x^2)^{-1/2}$$

$$= \left[4\left(1-\frac{3}{4}x^2\right)\right]^{-1/2}$$

$$= 4^{-1/2}\left(1-\frac{3}{4}x^2\right)^{-1/2}$$

$$= \frac{1}{2}\left(1-\frac{3}{4}x^2\right)^{-1/2}$$

$$T_{r+1} = \frac{1}{2}\frac{\left[\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)\left(-\frac{1}{2}-2\right)\cdots\cdots\left(-\frac{1}{2}-r+1\right)\right]\left(-\frac{3}{4}\right)^r x^{2r}}{r!}$$

$$= \frac{1}{2}\frac{\left[\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)\cdots\cdots\left(\frac{-2r+1}{2}\right)\right]\left(-\frac{3}{4}\right)^r x^{2r}}{r!}$$

$$T_{5+1} = \frac{1}{2}\frac{\left[\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)\left(-\frac{7}{2}\right)\left(-\frac{9}{2}\right)\left(-\frac{1}{2}-5\right)\left(-\frac{1}{2}-5+1\right)\right]\left(-\frac{3}{4}\right)^r x^{2r}}{5!}$$

$$\Rightarrow \frac{1}{2}\frac{\left[(-1)^r\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)\left(\frac{5}{2}\right)\left(\frac{2r-1}{2}\right)\right](-1)^r 3^r x^{2r}}{r!\;4^r}$$

$$= \frac{1}{2}\frac{\left[(-1)^r\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)\left(\frac{5}{2}\right)\left(\frac{2r-1}{2}\right)\right](-1)^r 3^r x^{2r}}{r!\;4^r}$$

$$\left(\frac{15309}{524288}x^{10}\right)$$

$$= \frac{1}{2}\left[\frac{1\times3\times5\times\cdots(2r-1)}{r!\;2^2}\;\frac{3^r x^{2r}}{2^{2r}}\right]$$

$$T_{r+1} = \frac{1\times3\times5\cdots\cdots(2r-1)(3^r x^{2r}}{2^{3r+1}\;r!}$$

$$T_{5+1} = \frac{1\times3\times5\times\cancel{6}\times7\times\cancel{8}\times9\;\;3^5 x^{10}}{2^{16}\times5!}$$

$$= \frac{\cancel{11022480}}{65536\times120}$$

$$= \frac{\cancel{11022480}}{7864320}$$

Coefficient of x^6 is

$$2r = 6$$

$$\boxed{r=3}$$

$$T_4 = \cancel{\tfrac{1}{2}}\left[\frac{(1\times\cancel{3}\times5)(3^3)}{2^{10}\;\cancel{3}\times2}\right]$$

$$= \frac{27\times5}{1024\times2} = \frac{135}{20}$$

Find the coefficient of x^{10} in the expansion of $\left(\frac{1}{x}+2\sqrt{x}\right)^{-5/2}$

$$= \frac{1}{x^{-5/2}}\left[1+2x^{5/2}\right]^{-5/2}$$

$$= x^{5/2}\left[1+2x^{3/2}\right]^{-5/2}$$

$$= x^{5/2}\left[\frac{-5}{2}\left(\frac{-5}{2}-1\right)\left(-\frac{5}{2}-2\right)\cdots\cdots\left(-\frac{5}{2}-r+1\right)\right]$$

19/50

$(1+x)^n = 1+nx$

Q. If x is so small that its squares and higher powers are neglected, show that

$$\frac{(16+5x)^{1/2} - (27-4x)^{1/3}}{(6+5x)} = \frac{1}{6} - \frac{13}{1296}x$$

$$= \frac{(16)^{1/2}\left(1+\frac{5}{16}x\right)^{1/2} - (27)^{1/3}\left(1-\frac{4}{27}x\right)^{1/3}}{6\left(1+\frac{5}{6}x\right)}$$

$$= \frac{4\left(1+\frac{1}{2}\cdot\frac{5}{16}x\right) - 3\left(1-\frac{1}{3}\cdot\frac{4}{27}x\right)}{6\left(1+\frac{5}{6}x\right)}$$

$$= 4 + \frac{\cancel{4}\times 5}{2\times \cancel{16}_4}x - 3 + \frac{\cancel{3}\times 4}{\cancel{3}\times 27}x$$

Q. If x is so small that its ~~cube and~~ higher powers ~~are~~ neglected, show that

$$\sqrt{\frac{1-x}{1+x}} = 1-x+\frac{x^2}{2}$$

$$= \frac{(1-x)^{1/2}}{(1+x)^{1/2}}$$

2010

Arithemetic Progression

A.P
a = first term
d = common difference.
Then AP is
$a, a+d, a+2d, a+3d \ldots$
$d = t_2 - t_1 = t_3 - t_2 = t_4 - t_3$

n^{th} term or general term
$t_n = a + (n-1)d = l$

Sum of n^{th} terms
$S_n = \frac{n}{2}[2a + (n-1)d]$
$= \frac{n}{2}[a + l]$

Q. If p-times the p^{th} term of an AP is equal to q-times the q^{th} term, prove that $(p+q)^{th}$ term is zero.

$p(a+(p-1)d) = q[a+(q-1)d]$ (Given)

$p(a+(p-1)d) - q[a+(q-1)d] = 0$

LHS $a(p-q) + d(p^2 - p - q^2 + q) = 0$
$a(p-q) + d[(p^2 - q^2) - (p-q)] =$

$$a(p-q)+d\left[(p+q)(p-q)-(p-q)\right]=0$$

Dividing whole eqn by $(p-q)$

$$a+d\left[(p+q)-1\right]=0$$

$$a+\left[(p+q)-1\right]d=0.$$

$(p+q)^{th}$ term $= 0$

Q. If a, b, c be the p^{th} term, q^{th} and r^{th} term of an AP, show that
$$p(b-c)+q(c-a)+r(a-b)=0$$

$$a = A+(p-1)D$$
$$b = A+(q-1)D$$
$$c = A+(r-1)D$$

$$a-b = A+(p-1)D-\left[A+(q-1)D\right]$$
$$= \cancel{A}+(p-1)D-\cancel{A}-(q-1)D$$
$$= D\left[p-\cancel{1}-q+\cancel{1}\right]$$
$$= D(p-q)$$

Similarly
$$b-c = D(q-r)$$

Similarly
$$c-a = D(r-p)$$

$$p\left[D(q-r)\right] + q\left[D(r-p)\right] + r\left[D(p-q)\right] = 0$$

$$D\left[p(q-r) + q(r-p) + r(p-q)\right] = 0$$

$$D\left[\cancel{pq} - \cancel{pr} + \cancel{qr} - \cancel{pq} + \cancel{pr} - \cancel{qr}\right] = 0$$

$$D \neq 0$$

So Hence proved

Q. How many terms of the series $3 + \frac{10}{3} + \frac{11}{3} + \cdots\cdots$ must be taken in order that the sum may be 23?

$a = 3$

$n = ?$

$S_n = 23$

$d = \frac{11}{3} - \frac{10}{3}$

$= \frac{1}{3}$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$23 = \frac{n}{2}\left[6 + (n-1)\frac{1}{3}\right]$$

$$46 = 6n + \frac{n^2}{3} - \frac{n}{3}$$

$$138 = 18n + n^2 - n$$

$\frac{46}{3}$, 138

$$n^2 + 17n - 138 = 0$$

$$n = \frac{-17 \pm \sqrt{(17)^2 - 4(1)(-138)}}{2}$$

$$= \frac{-17 \pm \sqrt{289 + 552}}{2}$$

$$= \frac{-17 \pm \sqrt{841}}{2}$$

$$= \frac{-17 \pm 29}{2}$$

$$= \frac{12}{2}, \frac{-29-17}{2}$$

-23

~~$n^2 + 17n - 138 = 0$.~~

Q. The p^{th} term of an AP is x and q^{th} term is y. Show that $(p+q)^{th}$ term is

$$\frac{p+q}{2}\left[x+y+\frac{x-y}{p-q}\right]$$

$$t_p = a+(p-1)d = x$$
$$t_q = a+(q-1)d = y.$$

$$a+(p-1)d = x$$
$$a+(q-1)d = y$$
$$2a + (p-1)d + (q-1)d = x+y$$
$$2a + (p+q-2)d = x+y$$
$$2a + (p+q-1)d - d = x+y$$
$$2a + (p+q-1)d = x+y+d$$

~~$t_{p+q} = a+(p+q-1)d$~~

$$S_{p+q} = \frac{p+q}{2}\left[2a+(p+q-1)d\right]$$

$$= \frac{p+q}{2}\left[x+y+d\right]$$

$$= \frac{p+q}{2}\left[x+y+\frac{x-y}{p-q}\right]$$

Hence Proved.

$$a+(p-1)d = x$$
$$a+(q-1)d = y$$
$$d(p-1-q+1) = x-y$$
$$d = \frac{x-y}{p-q}$$

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Q. The sums of first p, q, r terms of an AP are a, b, c respectively

$$\frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0$$

$$S_p = \frac{p}{2}\left[2A+(p-1)D\right] = a$$

$$S_q = \frac{q}{2}\left[2A+(q-1)D\right] = b$$

$$S_r = \frac{r}{2}\left[2A+(r-1)D\right] = c$$

$$\frac{a}{p} = \frac{1}{2}\left[2A+(p-1)D\right]$$

$$\frac{b}{q} = \frac{1}{2}\left[2A+(q-1)D\right]$$

$$\frac{c}{r} = \frac{1}{2}\left[2A+(q-1)D\right]$$

$$\frac{1}{2}\left[2A+(p-1)D\right](q-r)+\frac{1}{2}\left[2A+(q-1)D\right](r-p)+\frac{1}{2}\left[2A+(r-1)\right](p-q)=0$$

$$\left[A+\frac{(p-1)}{2}D\right](q-r)+\left[A+\left(\frac{q-1}{2}\right)D\right](r-p)+\left[A+\left(\frac{r-1}{2}\right)D(p-q)\right]$$

$$= A\left(\cancel{q}-\cancel{r}+\cancel{r}-\cancel{p}+\cancel{p}-\cancel{q}\right)+\frac{D}{2}\left[\quad\right]$$

$= 0$ Hence P

Q. The sum of p terms of an AP is q and the sum of q term is p. Find the sum of (p+q) terms. Also show that the sum of (p−q) terms is $(p-q)\left(1+\frac{2q}{p}\right)$

$2a+(p-1)d$

$$S_p = \frac{p}{2}\left[2a+(p-1)d\right] = q$$

$$S_q = \frac{q}{2}\left[2a+(q-1)d\right] = p$$

$$\cancel{\frac{p}{2}}\left[2a+(p-1)d\right] = \frac{2q}{p}$$

$$\cancel{\frac{q}{2}}\left[2a+(q-1)d\right] = \frac{2p}{q}$$

$$d\left[p-1-q+1\right] = \frac{2q}{p} - \frac{2p}{q}$$

$$S_{p+q} = \frac{p+q}{2}\left[2a+(p+q-1)d\right]$$

$$= \frac{p+q}{2}\left[\frac{2q}{p}-(p-1)d+(p+q-1)d\right]$$

$$2a = \frac{2q}{p}-(p-1)d$$

$$= \frac{p+q}{2}\left[\frac{2q}{p}-\cancel{pd}+\cancel{d}+\cancel{pd}+qd-\cancel{d}\right]$$

$$= \frac{p+q}{2}\left(\frac{2q}{p}+qd\right)$$

$$d\left[p-\cancel{1}-q+\cancel{1}\right] = \frac{2q}{p}-\frac{2p}{q}$$

$$= \frac{p+q}{2}\left[\frac{2q}{p}+\cancel{q}\left(\frac{-2p-2q}{p\cancel{q}}\right)\right]$$

$$d\left[p-q\right] = \frac{2q^2-2p^2}{pq}$$

$$= \frac{p+q}{\cancel{2}}\left[\frac{\cancel{2q}-\cancel{2}p-\cancel{2q}}{\cancel{p}}\right]$$

$$d = \frac{2q^2-2p^2}{pq(p-q)}$$

$$= \frac{-2(q+p)(q-p)}{}$$

$$= -p-q$$

$$= -(p+q)$$

$$= \frac{-2(p^2-q^2)}{pq(p-q)}$$

$$S_{p-q} = \frac{p-q}{2}\left[2a + (p-q-1)\,d\right]$$

$$= \frac{p-q}{2}\left[\frac{2q}{p} - (p-1)d + (p-q-1)\cancel{\frac{-2(p+q)}{pq}}\,d\right]$$

$$= \frac{p-q}{2}\left[\frac{2q}{p} - p\cancel{d} + \cancel{d} + \cancel{pd} - qd - \cancel{d}\right]$$

$$= \frac{p-q}{2}\left[\frac{2q}{p} - qd\right]$$

$$= \frac{p-q}{2}\left[\frac{2q}{p} - q\left(\frac{-2(p-2q)}{p\cancel{q}}\right)\right]$$

$$= \frac{p-q}{2}\left[\frac{2q+2p+2q}{p}\right]$$

$$= p-q\left[\frac{\cancel{2}(2q+p)}{\cancel{2}p}\right]$$

$$= (p-q)\left(1+\frac{2q}{p}\right)$$

Q. the sum of n terms of two arithmetic progressions are in the ratio of $2n+4 : 3n+5$ find the ratio of their 15^{th} terms.

$$\frac{S_n}{S_n} = \frac{\cancel{n/2}\left[2A_1 + (n-1)d_1\right]}{\cancel{n/2}\left[2A_2 + (n-1)d_2\right]} = \frac{2n+4}{3n+5}$$

$$\frac{\cancel{2}\left(A_1 + \frac{(n-1)}{2}d_1\right]}{\cancel{2}\left[A_2 + \left(\frac{n-1}{2}\right)d_2\right]} = \frac{2n+4}{3n+5}$$

$$t_{15} = \frac{a_1 + 14d_1}{a_2 + 14d_2}$$

$$\frac{n-1}{2} = 14$$

$$n - 1 = 28$$

$$\boxed{n = 29}$$

$$\frac{A_1 + \left(\frac{29-1}{2}\right)d_1}{A_2 + \left(\frac{29-1}{2}\right)d_2} = \frac{58+4}{87+5} = \frac{\cancel{62}\ 31}{\cancel{92}\ 46}$$

$$\frac{a_1 + 14d_1}{a_2 + 14d_2} = \frac{31}{46}$$

Q. How many terms of the series $10 + 9\frac{2}{3} + 9\frac{1}{3} - - - - -$ Should be taken to make the sum 155?

$a = 10 \qquad d = \frac{28}{3} - \frac{29}{3}$

$d = -\frac{1}{3}$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$155 = \frac{n}{2}\left[20 + (n-1)\left(-\frac{1}{3}\right)\right]$$

$$310 = 20n - \frac{n^2}{3} + \frac{n}{3}$$

$$930 = 60n - n^2 + n$$

$$-n^2 + 61n - 930 = 0$$

$$n^2 - 61n + 930 = 0$$

$$n = 30, \ 31$$

Q. The sum of three no.s in AP is 24 and their product is 440. Find the number.

$a-d,\ a,\ a+d,\ \cancel{a+2d}$

$\cancel{S_3 = \frac{3}{2}\left[2A+(2D)\right]}$ $\quad a+a-\cancel{d}+a+\cancel{d}=24$

$3a = 24$

$a = 8$

$(a-d)(a)(a+d)=440$

$(a^2-d^2)(a)=440$

$(64-d^2)\,8=440$

$64-d^2 = \frac{\cancel{440}^{55}}{\cancel{8}}$

$64-d^2=55$

$d^2 = 64-55$

$d^2 = 9$

$\boxed{d=\pm 3}$

5, 8, 11

11, 8, 5

Q. The sum of three nos in AP is 21 and sum of whose squares 179. Find them.

$a-\cancel{d}+a+\cancel{a}+\cancel{d}=21$

$3a=21$

$\boxed{a=7}$

$(a-d)^2+a^2+(a+d)^2=179$

$a^2-\cancel{2ad}+d^2+a^2+a^2+d^2+\cancel{2ad}=179$

$3a^2+2d^2=179$

$147 + 2d^2 = 179$

$2d^2 = 179 - 147$

$2d^2 = 32$

$d^2 = \frac{32}{2} = 16$ $\boxed{d = \pm 4}$

3, 7, 11

11, 7, 3

Q. If a, b, c are in AP, show that $a^2(b+c)$, $b^2(c+a)$, $c^2(a+b)$ are also in AP.

$2b = a + c$ $c - b = b - a$

$c^2(a+b) - b^2(c+a) = b^2(c+a) - a^2(b+c)$

$2b^2(c+a) = a^2(b+c) + c^2(a+b)$

$ab^3 =$

$\cancel{(c-b)}$ $[$ $c^2a + c^2b - b^2c - b^2a$

$= b^2c + b^2a - a^2b + a^2c$

$c(b^2 - a^2) + ab(b-a) = a(c^2 - b^2) + bc(c-b)$

$b-a\,[c(b+a) + ab] = c-b\,[a(c+b) + bc]$

$b-a\,[bc + ac + \cancel{ab}] = c-b\,[ac + \cancel{ab} + bc]$

$2b = a + c$

a, b c are in AP which is given

Hence Proved

2/11

G.P (Geometric Progression)

If a is the first term and r is common ratio then G.P is,

$a, ar, ar^2, ar^3, ar^{}$

Common ratio

$$\frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_4}{t_3}$$

n^{th} term or general term

$$t_n = ar^{n-1} = l.$$

Sum of n^{th} term

$$S_n = \frac{a(r^n - 1)}{r-1}, r > 1$$

$$S_n = \frac{a(1-r^n)}{(1-r)}, r < 1.$$

Q. If p, q, r are in AP, show that p^{th} ~~term~~, q^{th}, r^{th} terms of a GP are themselves in G.P.

Let A be the first term & R be common ratio

$t_p = AR^{p-1}$

$t_q = AR^{q-1}$

$t_r = AR^{r-1}$

p^{th}, q^{th} and r^{th} term are in GP if $\dfrac{t_q}{t_p} = \dfrac{t_r}{t_q}$

$(t_q)^2 = t_r \cdot t_p$

$\cancel{A}^2 R^{2q-2} = \cancel{A} R^{r-1} \cdot \cancel{A} R^{p-1}$

$R^{2q-2} = R^{(r-1)(p-1)}$ $\qquad R^{2q-2} = R^{p+r-2}$

$\cancel{2q-2 = pr - r - p + 1}$

$2q - \cancel{2} = p + r - \cancel{2}$

$\boxed{2q = p + r}$

This means p, q, r are in AP which is given
Hence proved

Q. The third term of a GP is $5\frac{4}{9}$ and the 7^{th} term is reciprocal of third term. Find the 5^{th} term.

$t_3 = ar^2 = \dfrac{49}{9}$

$t_7 = \dfrac{1}{ar^2} = ar^6$ $\qquad \cancel{a^2 r^8 = \frac{9}{49}}$

$ar^6 = \dfrac{9}{49}$

$\dfrac{ar^{\cancel{6}\,4}}{\cancel{ar^2}} = \dfrac{9}{49} \times \dfrac{9}{49}$

$r^4 = \left(\dfrac{9}{49}\right)^2$

$r^4 = \left(\dfrac{3}{7}\right)^4$ $\qquad \boxed{r = \dfrac{3}{7}}$

ar^2

$\dfrac{49}{9} = a \dfrac{9}{49}$

$a = \left(\dfrac{49}{9}\right)^2$

$T_5 = ar^4$

$= \left(\dfrac{49}{9}\right)^2 \times \left(\dfrac{9}{49}\right)^2 = 1$

I Q. The first and second terms of a GP are 192 and -48 respectively and the last term is $\frac{-3}{256}$. Find the no. of terms

$a = 192$

$ar^1 = -48$

$r = \frac{-48}{192}$

$l = ar^{n-1}$

$t_n = (192)\left(\frac{-48}{192}\right)^{n-1}$

$\frac{-3}{256} = 192\left(\frac{-48}{192}\right)^{n-1}$

$\frac{-48}{192 \times 192} \times \frac{-3}{256} = \left(\frac{-48}{192}\right)^{n}$

$\frac{-1}{16384} = \left(\frac{-48}{\cancel{192}\,4}\right)^{n-1}$

$n = 8$

$\left(-\frac{1}{4}\right)^7 = \left(-\frac{1}{4}\right)^{n-1}$

$n - 1 = 7$

$\boxed{n = 8}$

Q.

Q. If x, y, z respectively be the p^{th}, q^{th}, r^{th} terms of a GP, show that

$x^{q-r}\, y^{r-p}\, z^{p-q} = 1$

Let A is the first term and R is the common ratio

$t_p = AR^{p-1} = x$

$t_q = AR^{q-1} = y$

$t_r = AR^{r-1} = z$

$$x = AR^{p-1}$$
$$x^{q-r} = A^{q-r} R^{(q-r)(p-1)}$$
$$y^{r-p} = A^{r-p} R^{(r-p)(q-1)}$$
$$z^{p-q} = A^{p-q} R^{(p-q)(r-1)}$$

$$\cancel{x^{q-r} y^{r-p} z^{p-q} = A^{(q-r)(r-p)(p-q)}}$$
$$x^{q-r} y^{r-p} z^{p-q} = A^{\cancel{q}-\cancel{r}+\cancel{r}-\cancel{p}+\cancel{p}-\cancel{q}}$$
$$= R^{(pq-}$$

$$x^{q-r} y^{r-p} z^{p-q} = 1.$$

Q. In a G.P, the first term is 7, the last term is 448 and the sum is 889. Find the common ratio and the series.

$$S_n = \frac{a(r^n-1)}{r-1} \qquad l = ar^{n-1}$$
$$S_n = \frac{ar^n - a}{r-1} \qquad = ar^n \cdot r^{-1}$$
$$= \frac{ar^n}{r}$$
$$= \frac{lr - a}{r-1} \qquad ar^n = lr$$

$$889 = \frac{448r - 7}{r-1}$$

$r=2$

$$889 = \frac{448r-7}{r-1} \qquad \cancel{r-1 = \frac{441}{889}}$$

$a=7$

7, 14

Q. $7+77+777+\ldots$ to n terms

let $S_n = 7+77+777+\ldots$ to n terms

$= 7(1+11+111+\ldots$ to n terms$)$

$= \frac{7}{9}(9+99+999+\ldots$ to n terms$)$

$= \frac{7}{9}[(10-1)+(10^2-1)+(10^3-1)+\ldots$ n terms$]$

$= \frac{7}{9}[10+10^2+10^3+\ldots$ n term$] - [1+1+1+1\ldots$ n$]$

$$= \frac{7}{9}\left[\frac{10(10^n-1)}{10-1} - n\right]$$

$$= \frac{7}{9}\left[\frac{10^{n+1}-10-9n}{9}\right]$$

$$= \frac{9}{81}\left[10^{n+1}-10-9n\right]$$

Q. $.5+.55+.555+.5555+\ldots$ to ∞

$= 5(0.1+0.11+0.111+\ldots n)$

$= \frac{5}{9}(0.9+0.99+.999\ldots+n)$

$$= \frac{5}{9}\left(\frac{9}{10}+\frac{99}{100}+\frac{999}{1000}+\ldots n\right)$$

$$= \frac{5}{9}\left[\left(1-\frac{1}{10}\right)+\left(1-\frac{1}{10^2}\right)+\left(1-\frac{1}{10^3}\right)+\left(1-\frac{1}{10^4}\right)\ldots\right]_n$$

$$= \frac{5}{9}\left[(1+1+1+1 \ldots\ldots n) - \left(\frac{1}{10} + \frac{1}{10^2} + \frac{1}{10^3} \ldots \right) \right]$$

$=$

Q. How many terms of a series $\frac{2}{9} - \frac{1}{3} + \frac{1}{2} + \ldots$ must be taken to amount to $\frac{55}{72}$?

$$\frac{\frac{1}{2}}{-\frac{1}{3}} = \frac{-\frac{1}{3}}{\frac{2}{9}}.$$

$$\frac{1}{2} \times -3 = \frac{-1}{\cancel{3}} \times \frac{\cancel{9}^{3}}{2}$$

$$-\frac{3}{2} = -\frac{3}{2}.$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{\frac{2}{9}\left[1 - \left(\frac{-3}{2}\right)^n\right]}{1 + \frac{3}{2}}$$

$$\frac{55}{72} = \frac{4}{45}\left[1 - \left(\frac{-3}{2}\right)^n\right]$$

$$\frac{a(1-r^n)}{1-r}$$

$$\frac{55\times 45}{4\times 72} = \left[1-\left(\frac{-3}{2}\right)^n\right]$$

$$-\left(\frac{-3}{2}\right)^n = \frac{2475}{288} - 1$$

$$\cancel{\left(\frac{-3}{2}\right)^n = -\frac{3672}{288}} \qquad \left(\frac{-3}{2}\right)^n = \frac{-2187}{288}$$

$$\cancel{\left(\frac{-3}{2}\right)^n = -\frac{21}{4}} \qquad \left(\frac{-3}{2}\right)^n = \frac{-243}{32}$$

$$\left(\frac{-3}{2}\right)^n = \left(\frac{-3}{2}\right)^5$$

$$\boxed{n=5}$$

Q. Find three no-s in G.P whose sum is 21 and whose product is 216.

Let $\frac{a}{r}, a, ar$ are three no. in G.P.

$$\frac{a}{r}\cdot a\cdot ar = 216$$

$$a^3 = 216$$

$$\boxed{a = \pm 6}$$

$$\frac{a}{r} + a + ar = 21$$

$$a(1+r+r^2) = 21r.$$

$$1+r+r^2 = \frac{21r}{a}$$

$$1+r+r^2 = \frac{21\,7r}{6\,2}$$

$$r^2+r-\frac{7}{2}r+1=0$$

Q. Find three numbers in GP whose sum is 19 and the sum of whose squares is 133

$a + ar + ar^2 = 19$

$a^2 + a^2r^2 + a^2r^4 = 133$

$(a+ar+ar^2) = 361$

$(a+ar)^2 + a^2r^4 + 2(a+ar)(ar^2) = 361$

$a^2 + a^2r^2 + 2a^2r + a^2r^4 + 2a^2r^2 + 2a^2r^3 = 36$

$a^2 + a^2r^2 + a^2r^4 + 2ar(a+ar+ar^2) = 361$

$133 + 2ar(19) = 361$

$\cancel{2ar \times 19 = \frac{361}{133}}$ $\qquad 38ar = 361 - 1$

$\cancel{2ar = \frac{361}{133 \times 19}}$

$\cancel{ar = \frac{1}{7}}$

$\cancel{AA} \; \frac{1}{14}$

Q. If a, b, c, d are in G.P, show that a^2+b^2, b^2+c^2, c^2+d^2 are also in G.P

$$\frac{b^2+c^2}{a^2+b^2} = \frac{c^2+d^2}{b^2+c^2}$$

$\frac{c}{b} = \frac{b}{a}$

$b^2 = ac$

$$(b^2+c^2)^2 = (a^2+b^2)(c^2+d^2)$$

$$(a^2r^4 + a^2r^6)^2$$

$$= (a^2r^2 + a^2r^4)(a^2r^6 + a^2r^8)$$

$[c(c+a)]^2$

$$= (a^4r^8 + a^4r^{10} + a^4r^{10} + a^4r^{12})$$

$a = AR$

$b = AR^2$

$c = AR^3$

$d = AR^4$

$$a^4r^8 + a^4r^{12} + 2a^4r^{10} = a^4r^8 + 2a^4r^{10} + a^4r^{12}$$

LHS = RHS.

II nd Method

LHS $$\frac{A^2R^4 + A^2R^6}{A^2R^2 + A^2R^4}$$

RHS $$= \frac{A^2R^6 + A^2R^8}{A^2R^4 + A^2R^6}$$

$$= \frac{\cancel{A^2}(R^4+R^6)}{\cancel{A^2}(R^2+R^4)}$$

$$= \frac{\cancel{A^2R^2}(R^3+R^4)}{\cancel{A^2R^2}(R^2+R^3)}$$

$$= \frac{R^2(R^2+R^3)}{R^2(1+R^2)} = \frac{R^2+R^3}{1+R^2}$$

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Vectors

Scalar and vector.

$\vec{a}$ and $\vec{b}$ are two vectors

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.

Unit vector of $\vec{a} = \dfrac{a_1\hat{i} + a_2\hat{j} + a_3\hat{k}}{\sqrt{a_1^2 + a_2^2 + a_3^2}}$

$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3 k$

$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$

Dot product is commutative.

$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$

Vector product (cross product)

$\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin\theta \cdot \hat{\eta}$, $\hat{n}$ is a unit vector perpendicular to both the vector.

$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

Q. Find the angle between the vectors $2\hat{i}+6\hat{j}+3\hat{k}$ and $12\hat{i}-4\hat{j}+3\hat{k}$.

$a \cdot b = |a||b| \cos\theta$.

$$\cos\theta = \frac{24-24+9}{\sqrt{4+36+9}\ \sqrt{144+16+9}}$$

$$= \frac{9}{\sqrt{49}\ \sqrt{169}}$$

$$= \frac{9}{7 \times 13}$$

$$\theta = \cos^{-1}\left(\frac{9}{91}\right)$$

$$\cos\theta = \frac{9}{91}$$

Q. Prove that $\hat{i}+2\hat{j}+8\hat{k}$ and $2\hat{i}+3\hat{j}-\hat{k}$ are perpendicular to each other.

$$a \cdot b = (\hat{i}+2\hat{j}+8\hat{k})(2\hat{i}+3\hat{j}-\hat{k})$$

$$= 2+6-8$$

$$= 8-8$$

$$= 0$$

$$a \cdot b = 0 \quad \therefore a \perp b$$

II nd

Q. The constant forces $2\hat{i}-5\hat{j}+6\hat{k}$, $-\hat{i}+2\hat{j}-\hat{k}$ and $2\hat{i}+7\hat{j}$ act on a particle which is displaced from position $4\hat{i}-3\hat{j}-2\hat{k}$ to position $6\hat{i}+\hat{j}-3k$. Find the total work done

Sol

$$d = (6\hat{i}+4\hat{j}-3k) - (4\hat{i}-3\hat{j}-2\hat{k})$$
$$= 2\hat{i}+4\hat{j}-\hat{k}$$

$$F = (2\hat{i}-5\hat{j}+6\hat{k}) + (-\hat{i}+2\hat{j}-\hat{k}) + (2\hat{i}+7\hat{j})$$
$$= 3\hat{i}+4\hat{j}+5\hat{k}.$$

$\hat{a} = \frac{a}{|a|}$

$$W = F \cdot d$$
$$= (3\hat{i}+4\hat{j}+5\hat{k}) \cdot (2\hat{i}+4\hat{j}-\hat{k})$$
$$= 6+16-5$$
$$= 22-5 = 17$$

Q. Forces of magnitudes 5 and 3 units acting in the directions $6\hat{i}+2\hat{j}+3\hat{k}$ and $3\hat{i}-2\hat{j}+6\hat{k}$ respectively act on a particle which is displaced from the point $(2, 2, -1)$ to $(4, 3, 1)$. Find the work done by the force.

$$d = 4\hat{i}+3\hat{j}+\hat{k} - (2\hat{i}+2\hat{j}-\hat{k})$$
$$= 2\hat{i}+\hat{j}+2\hat{k}$$

First force of magnitude 5 unit acting in the direction $6\hat{i}+2\hat{j}+3k = 5\frac{6\hat{i}+2\hat{j}+3k}{\sqrt{36+4+9}}$

$\sin A - \sin B = 2 \sin \frac{C+D}{2} \sin \frac{C-D}{2}$

~~$\sin A + \sin B$~~ $= 2 \sin \frac{C+D}{2} \sin\left(\frac{D-C}{2}\right)$

$\cos A - \cos B =$

$$= 5\left(\frac{6\hat{i} + 2\hat{j} + 3\hat{k}}{7}\right)$$

$$= \frac{30}{7}\hat{i} + \frac{10}{7}\hat{j} + \frac{15}{7}\hat{k}.$$

Second force $= 3\left(\frac{3\hat{i} - 2\hat{j} + 6\hat{k}}{\sqrt{9+4+36}}\right)$

$$= 3\left(\frac{3\hat{i} - 2\hat{j} + 6\hat{k}}{7}\right)$$

$$= \frac{9}{7}\hat{i} - \frac{6}{7}\hat{j} + \frac{18}{7}\hat{k}.$$

Total $F = F_1 + F_2$

Q $= \frac{39}{7}\hat{i} + \frac{4}{7}\hat{j} + \frac{33}{7}\hat{k}.$

$W = F \cdot d$

$$= (2\hat{i} + \hat{j} + 2\hat{k})\left(\frac{39}{7}\hat{i} + \frac{4}{7}\hat{j} + \frac{33}{7}\hat{k}\right)$$

$$= \frac{78}{7} + \frac{4}{7} + \frac{66}{7}$$

$$= \frac{78+70}{7} = \frac{148}{7}.$$

CON

Q.

continued from schoolnote

Differentiation

$\frac{a}{b} \div \frac{a}{b} \times \frac{b}{a}$ $\cot x = -\operatorname{cosec}^2 x$

$h \to \Delta x$

Q. $\tan x$.

$$\frac{\Delta y}{\Delta x} \lim_{h \to 0} \frac{\tan(x+h) - \tan x}{h}$$

$$= \lim_{h \to 0} \frac{\frac{\sin(x+h)}{\cos(x+h)} - \frac{\sin x}{\cos x}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{\sin(x+h)\cos x - \sin x \cos(x+h)}{\cos(x+h) \cdot \cos x}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{\sin(\cancel{x}+h-\cancel{x})}{\cos(x+h) \cdot \cos x}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{\sin h}{\cos(x+h) \cdot \cos x}}{h}$$

$$= \lim_{h \to 0} \frac{1}{\cos(x+h) \cdot \cos x}$$

$$= \lim_{h \to 0} \frac{1}{\cos^2 x}$$

$$= \sec^2 x.$$