4/Aug/2016

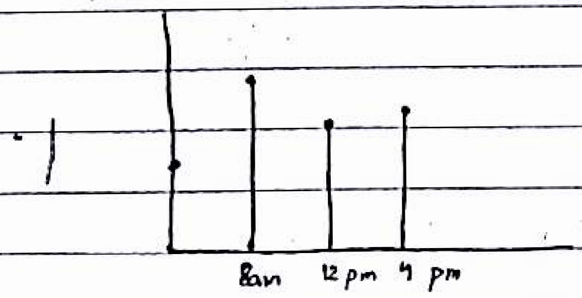
Signals & Systems

Signal :- A signal is defined as a physical quantity that varies with time or space or any other independent variable.
A signal is a function representing a physical quantity or variable and it contains information about the behaviour or nature of the phenomena.
Mathematically a signal is represented as a function of an independent variable 't'. Usually 't' represents time. A signal is therefore represented by $x(t)$.

E.g. 1) AC Power Supply
2). ECG
3) Variation of room temp.

* Continuous & Discrete Signals

A signal $x(t)$ is a continuous time signal if 't' is a continuous variable in time domain.
If 't' is a discrete variable i.e. $x(t)$ is defined at discrete time when $x(t)$ is a discrete time signal.



Since a discrete type signal is defined at discrete times, a discrete time signal is often identified as a sequence of signals denoted by $x[n]$ where n is integer.

→ Real & complex signals

A signal $x(t)$ is a real signal if its value is a complex no.

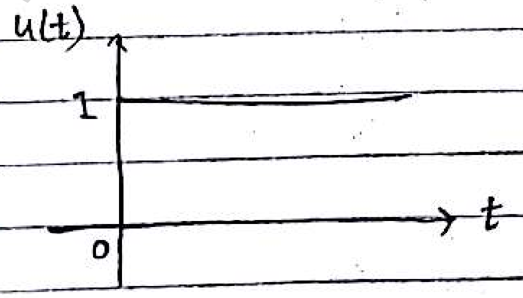
A complex no. is represented as:-

$$x(t) = x_1(t) + j\,x_2(t)$$

* Basic Continuous Time Signals

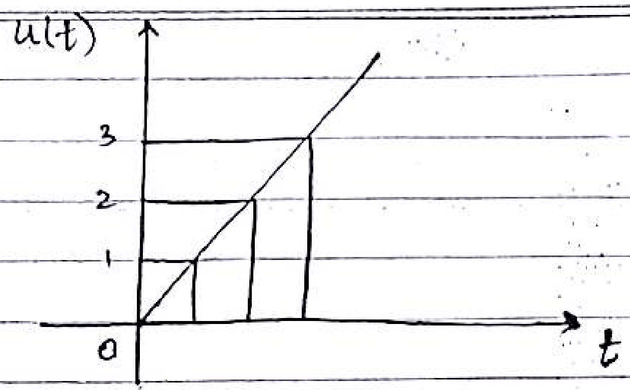
1.) Unit Step Signal :- It is defined as

$$u(t) = 1 \quad \text{for } t \geq 0$$
$$u(t) = 0 \quad \text{for } t < 0$$



2.) Unit Ramp Signal :- It is defined as

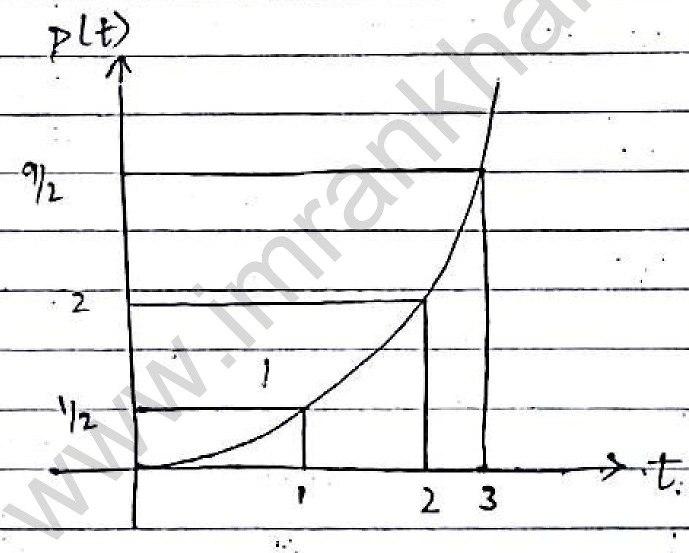
$$r(t) = t \quad \text{for } t \geq 0 \quad \text{or } t \cdot u(t)$$
$$= 0 \quad \text{for } t < 0$$



3) Unit Parabolic Signal

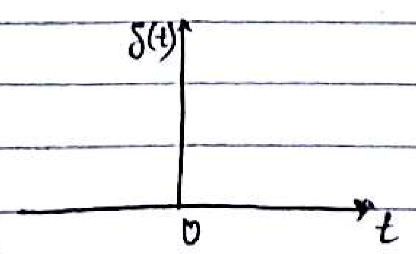
$$p(t) = \frac{t^2}{2} \quad \text{for } t \geq 0 \; :$$

$$p(t) = 0 \quad \text{for } t < 0$$



4) Unit Impulse Signal

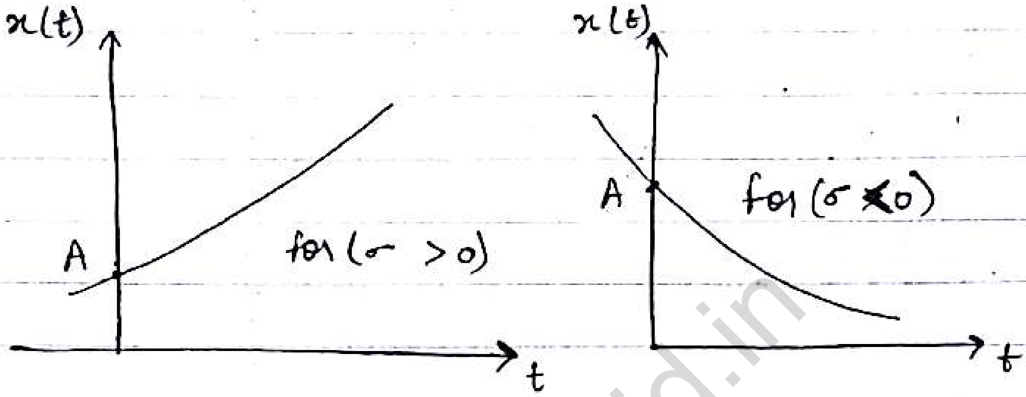
$$\delta(t) = \infty \quad \text{for } t = 0$$
$$\delta(t) = 0 \quad \text{for } t \neq 0$$



5.) Real Exponential Signal :-

~~x(t) =~~

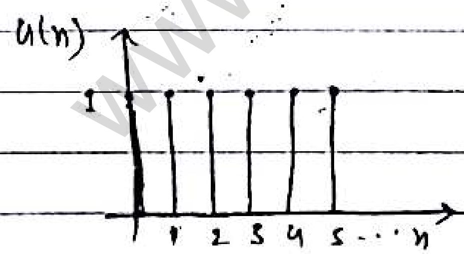
$$x(t) = A e^{\sigma t}$$



* Basic Discrete Time Signals

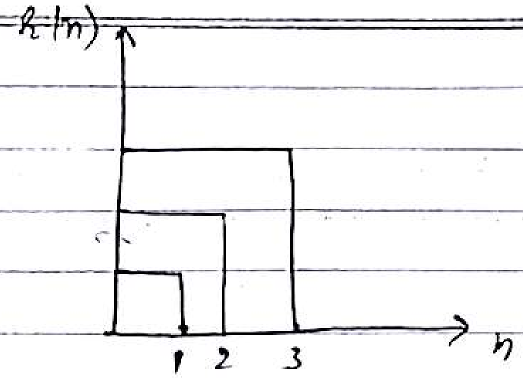
1.) Unit Step Sequence

$$u(n) = 1 \quad \text{for } n \geq 0$$
$$u(n) = 0 \quad \text{for } n < 0$$



2.) Ramp Sequence

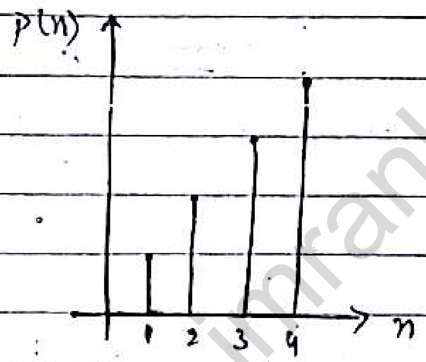
$$r(n) = n \quad \text{for } n \geq 0$$
$$r(n) = 0 \quad \text{for } n < 0$$



3.) Unit Parabolic Sequence

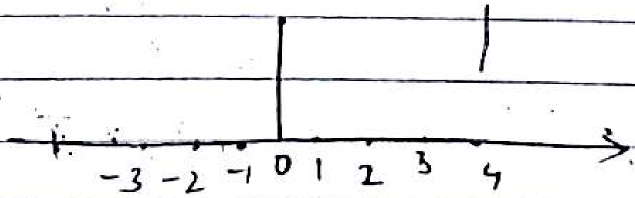
$$p(n) = \frac{n^2}{2} \quad \text{for } n \geq 0$$

$$p(n) = 0 \quad \text{for } n < 0$$



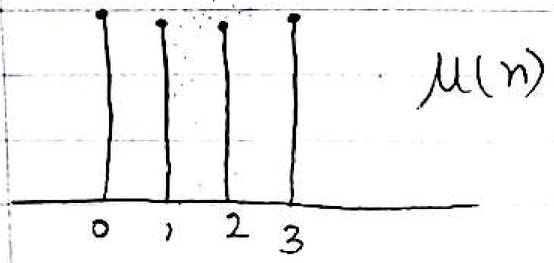
4.) Unit Impulse Sequence

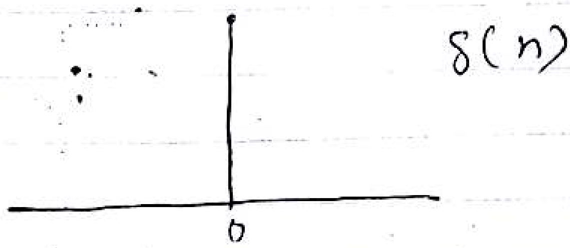
$$\delta[n] = 1 \quad \text{for } n = 0$$
$$\delta(n) = 0 \quad \text{for } n \neq 0$$

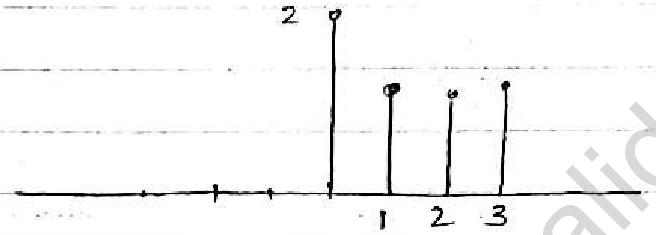


Plot

$$\mu(n) + \delta(n)$$







Basic Operations on Signals

1.) Time Shifting :- The type shifting of $x(t)$ or $x(n)$ may delay or advance the signal in ~~the signal in~~ time. Mathematically this can be represented by:-

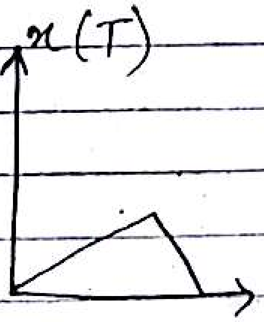
$$y(t) = x(t - T)$$

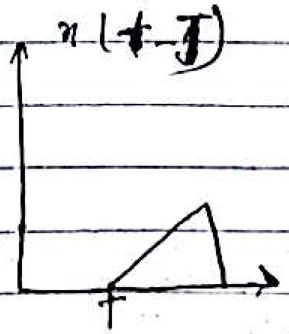
or

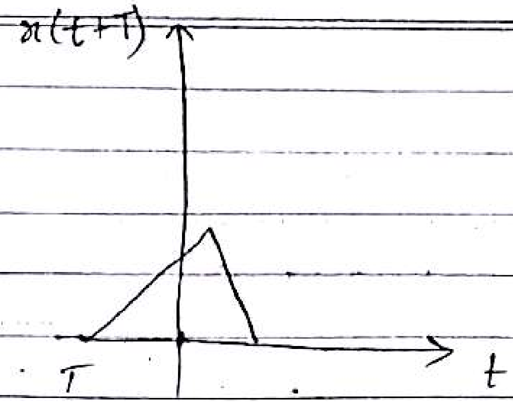
$$y[n] = x(n - n)$$

→ If T or t is +ve, the shifting delays the signal.

→ If T or t is −ve, the shifting advances the signal



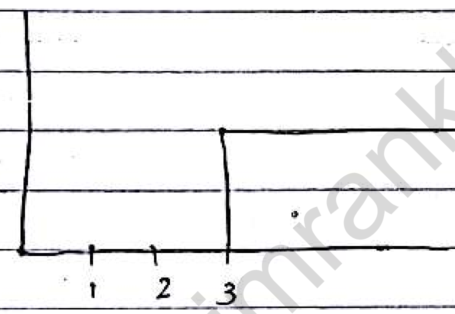




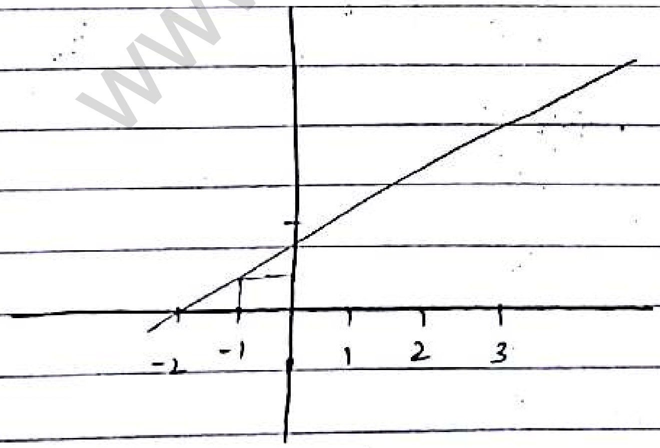
Q. Plot the following signals:-

1.) $u(t-3)$
2.) $x(t+2)$
3.) $u[n-2] + u[n+1]$

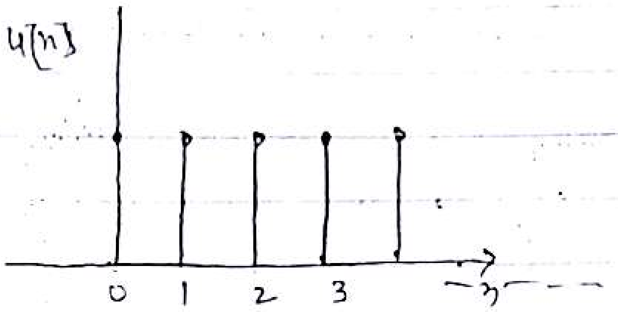
①

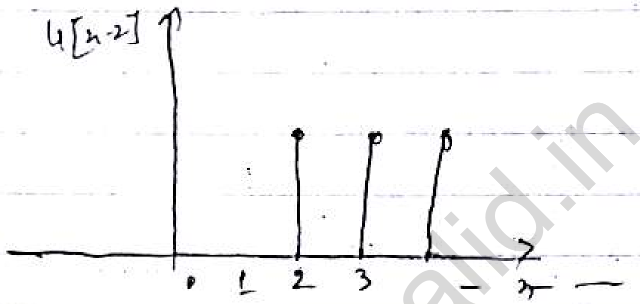


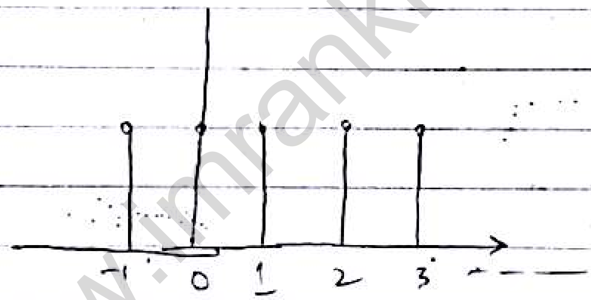
②

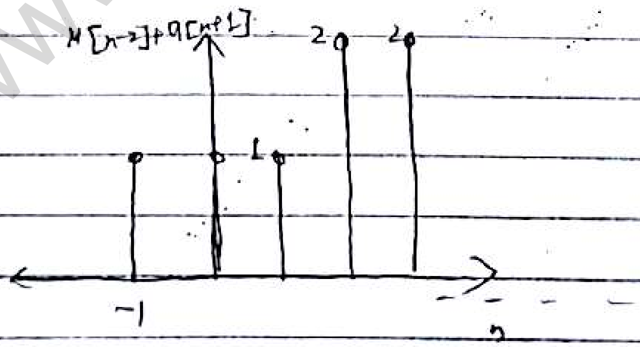


③ $u[n-2] + u[n+1]$







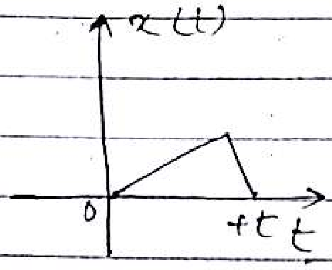


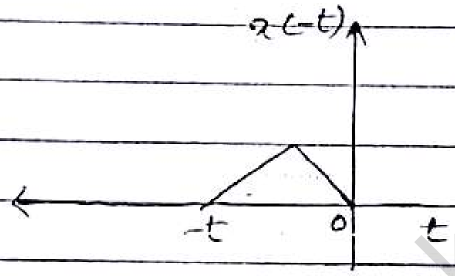
Time Reversal

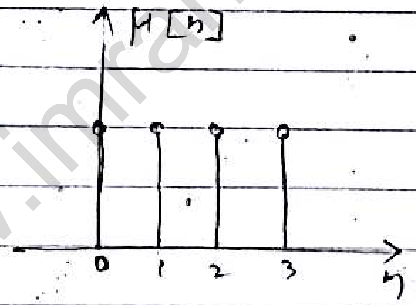
The time reversal of signal $x(t)$ can be obtained by holding the signal about $t=0$ which is denoted by

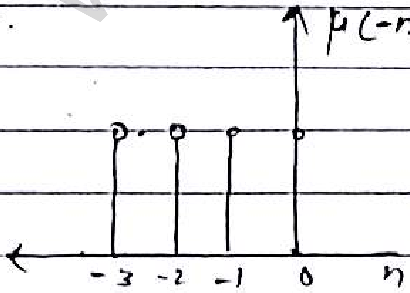
$$x(-t)$$

Examples

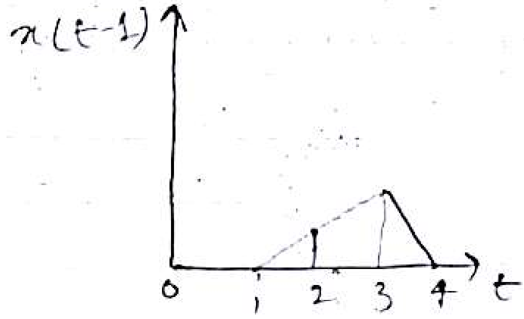




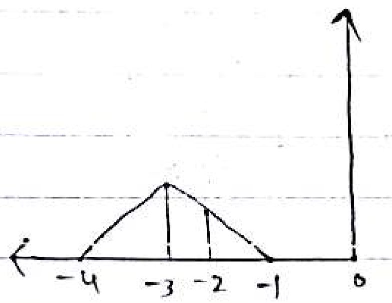


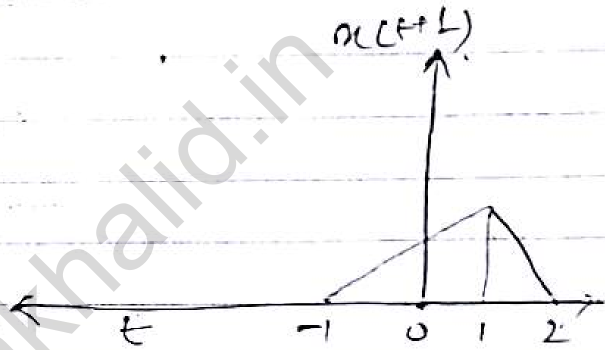


$x[t-1)$



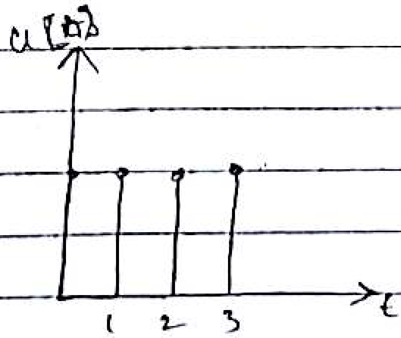
$x(-t+1)$

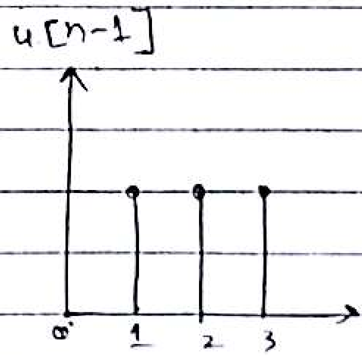


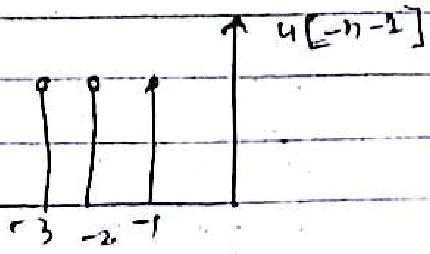


State the following signals

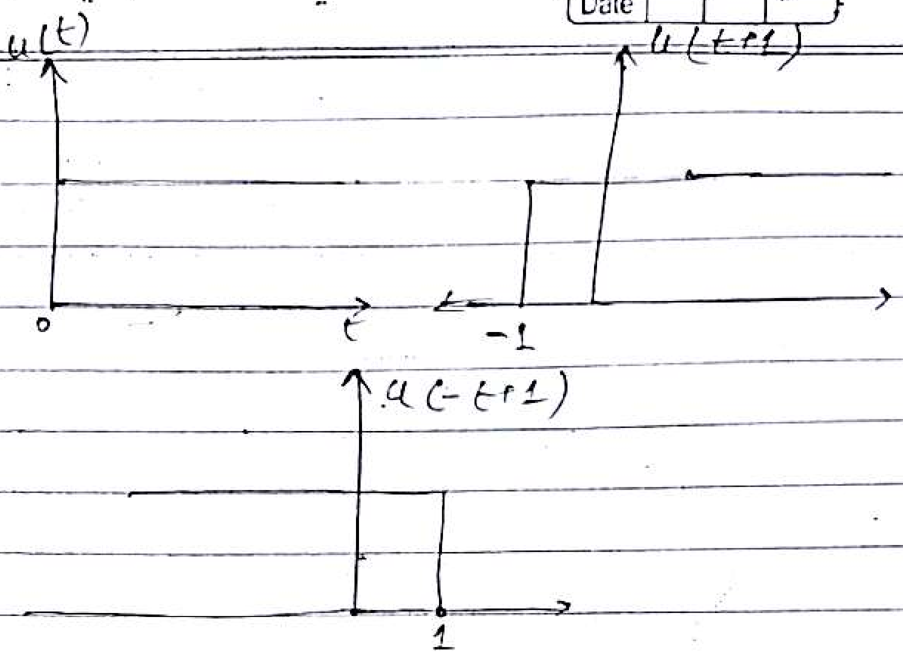
1. $u(-t+1)$
2. $r(-t+2)$
3. $u[-n+2] \cdot u[n]$
4. $u[-n-1]$







①



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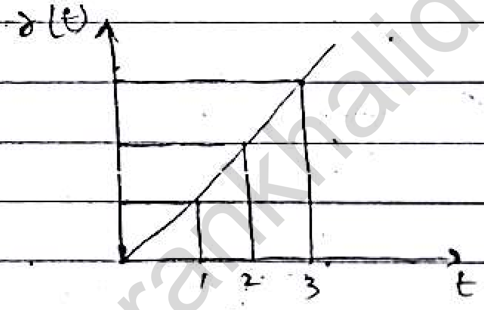
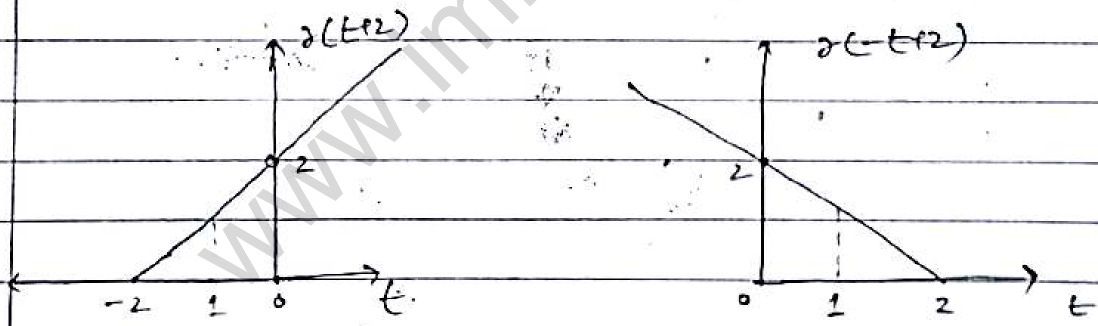
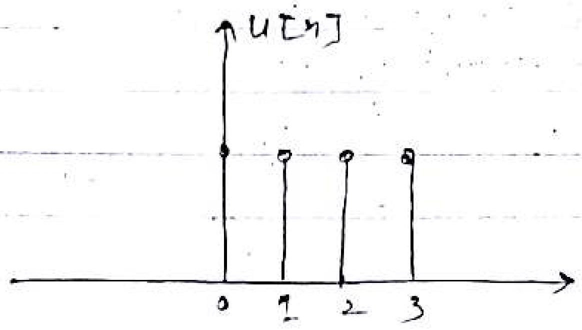
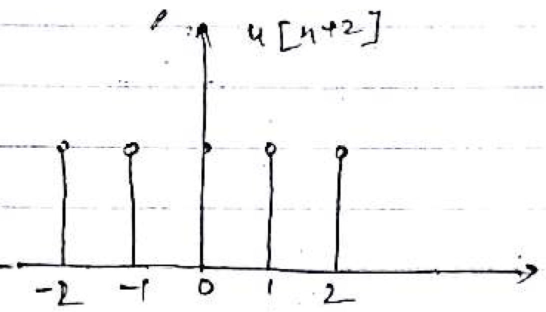


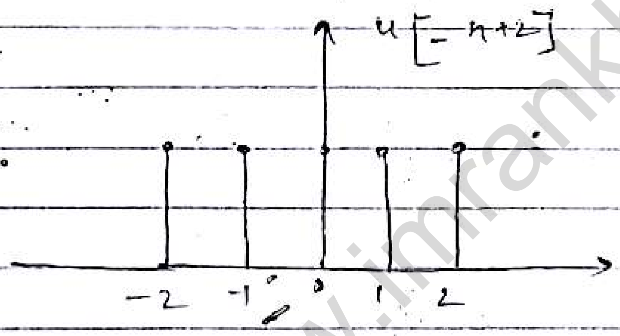


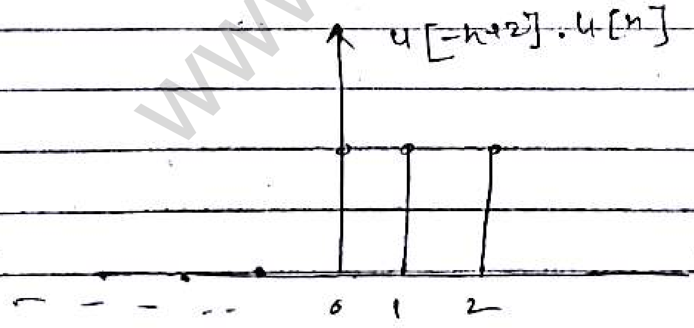
③

$u[-n+2] \cdot u[n]$

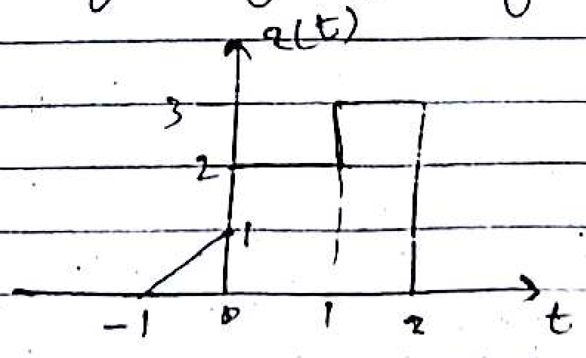








Sketch the following signal for given $x(t)$ and $x[n]$

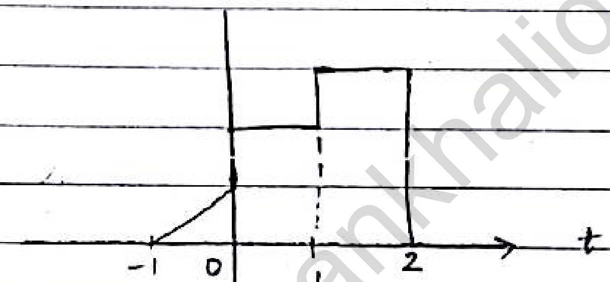


① $x(t) \cdot u(1-t)$

② $x(t) \cdot u(t-\frac{3}{2})$

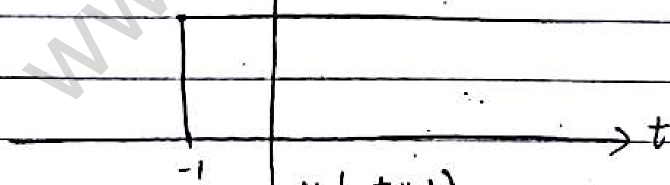
$x(t)$	$x(-t)$
$(x)(t-T) \Rightarrow$	$x(-t-T) \Leftarrow$
$x(t+T) \Leftarrow$	$x(-t+T) \Rightarrow$

1) $x(t) \cdot u(1-t)$

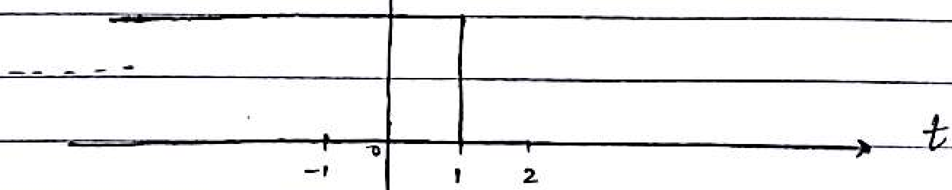


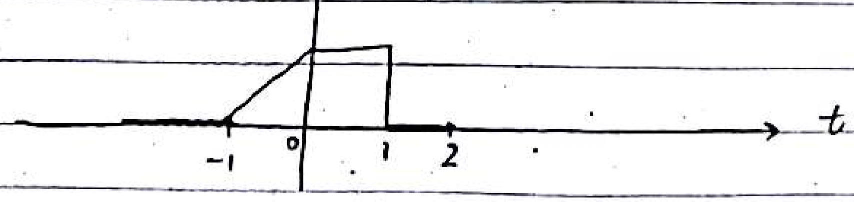
$u(t)$

$u(t+1)$

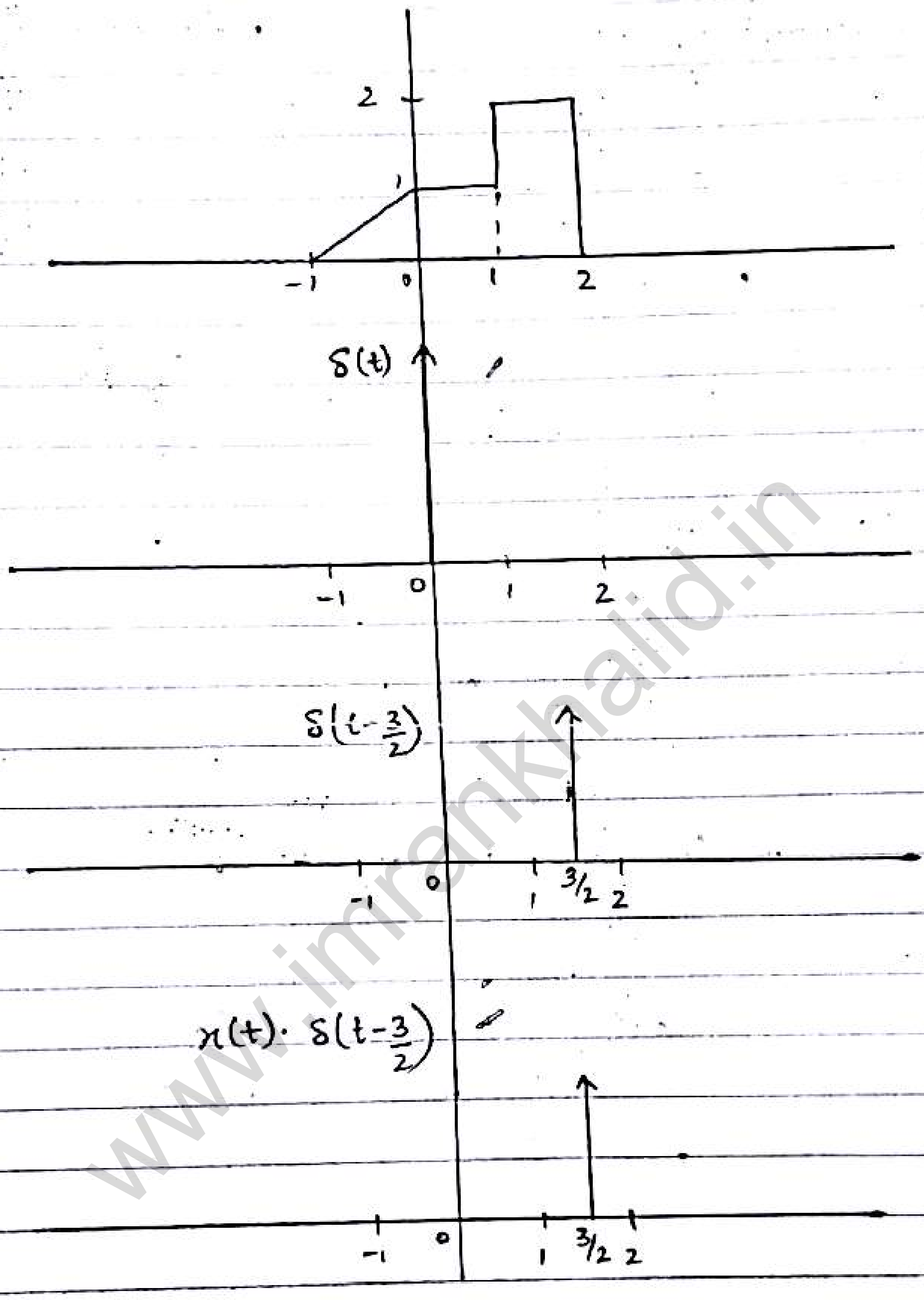


$u(-t+1)$

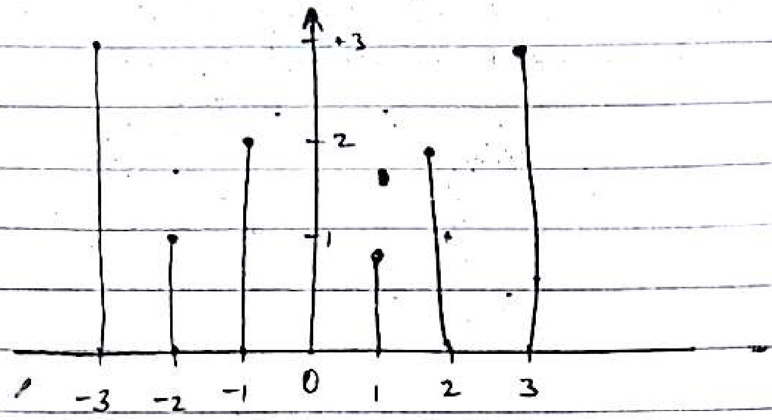




2. $x(t) \cdot \delta\left(t - \frac{3}{2}\right)$

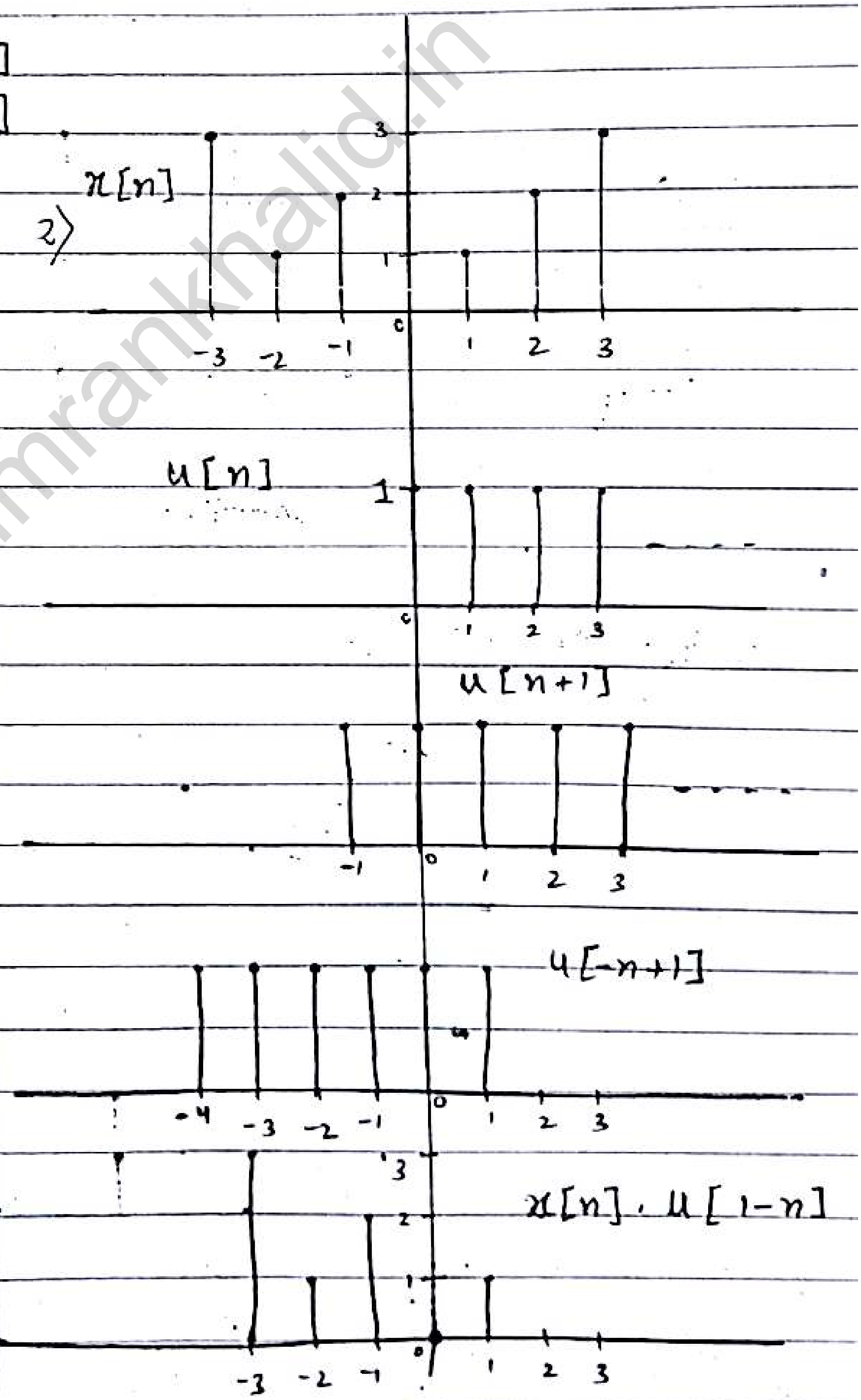


Q. Sketch the following signal for given $x[n]$

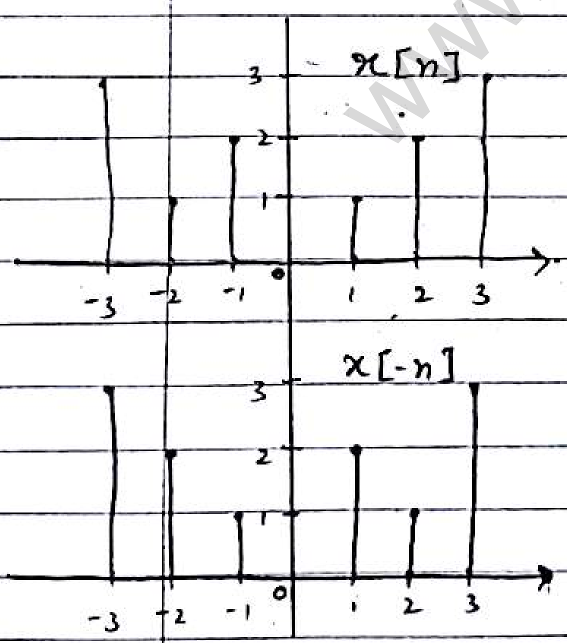


1.) $x[-n]$
2.) $x[n] \cdot u[1-n]$
3.) $x[n] \cdot \delta[n-1]$
4.) $x[n+2]$
5.) $x[2-n]$

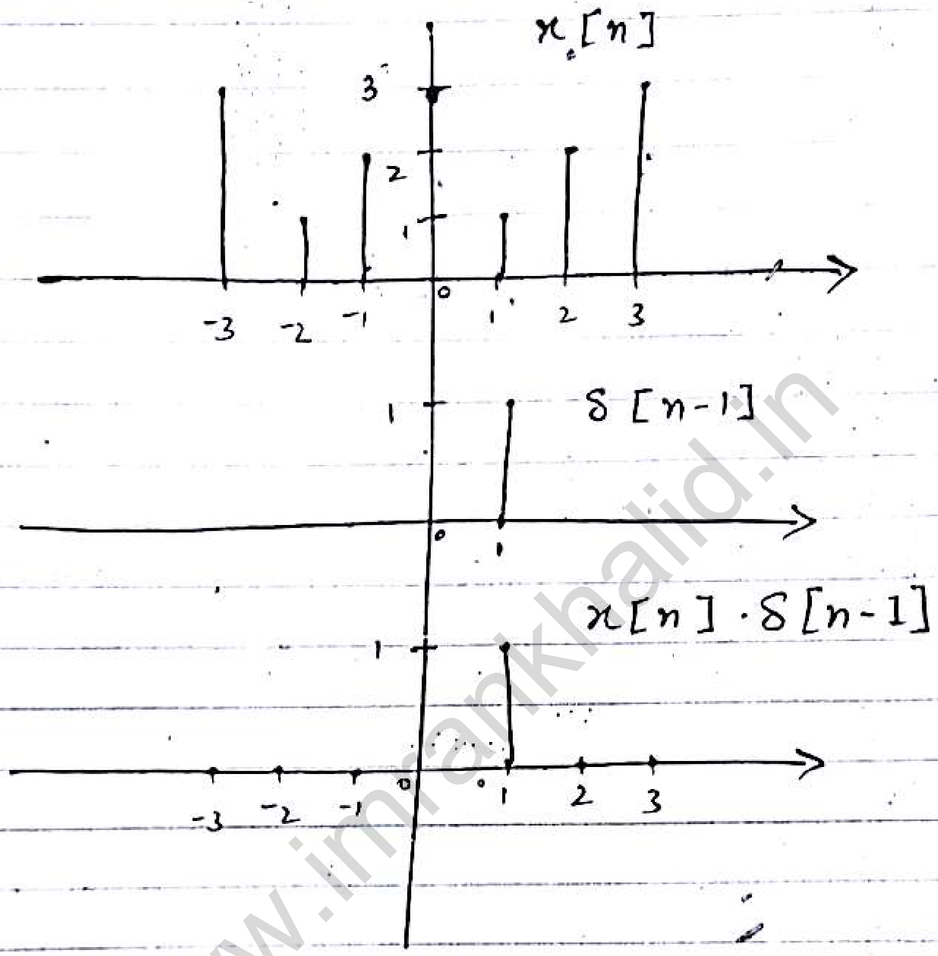
2) $x[n] \cdot u[1-n]$



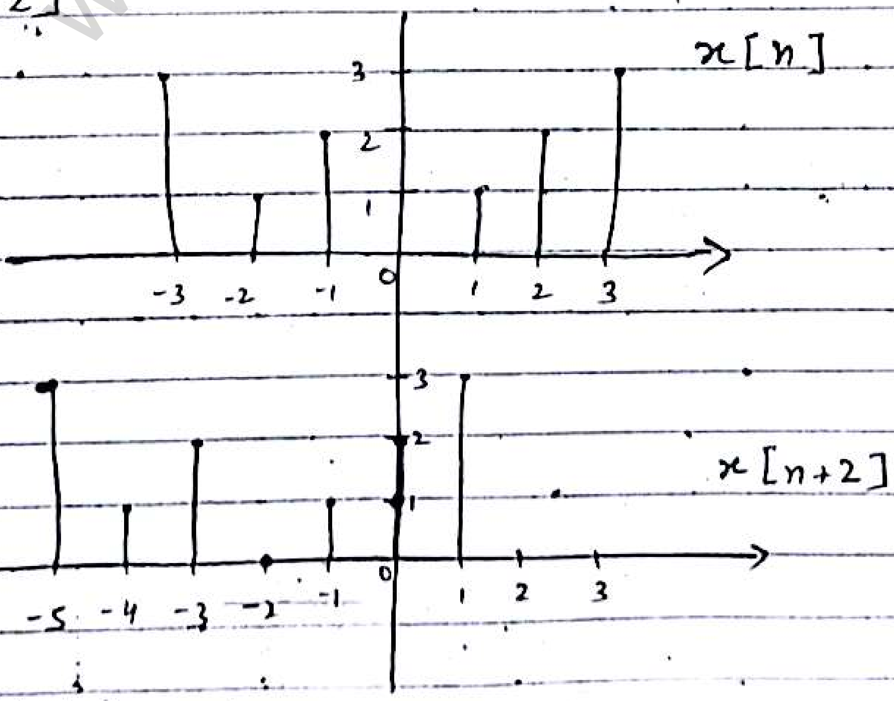
1.) $x[-n]$



3) $x[n] \cdot \delta[n-1]$.



4.) $x[n+2]$



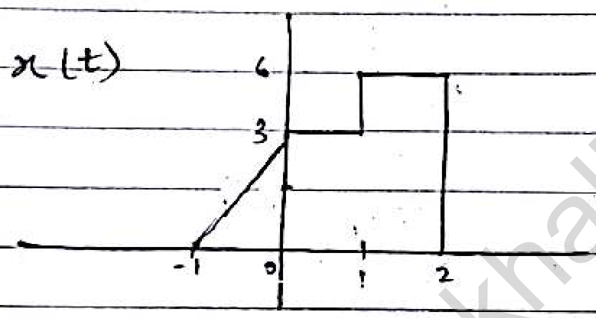
5.) → 4^{th} mirror image

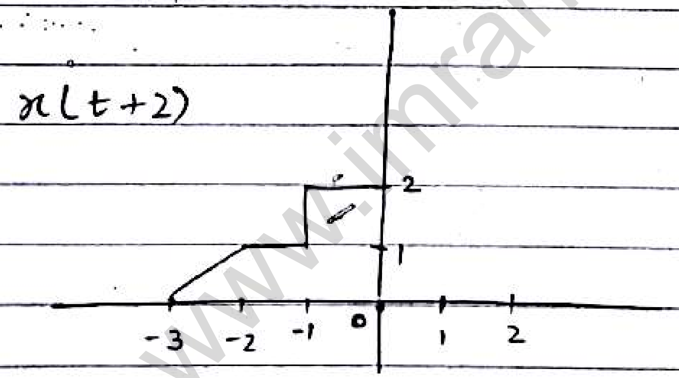
* Amplitude Scaling

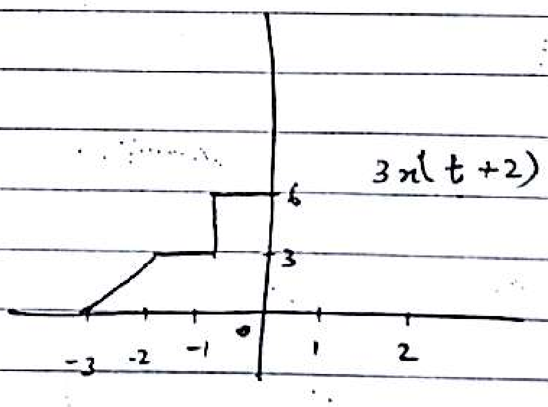
Amplitude scaling can be represented as follows:-

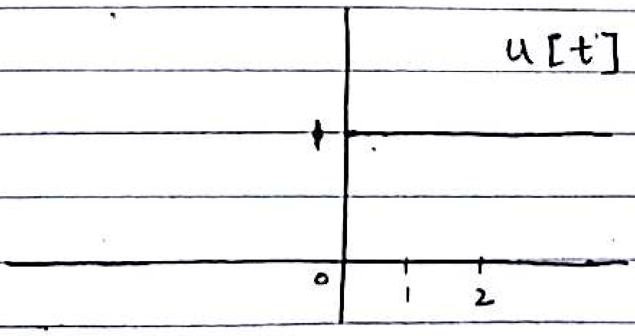
$$y(t) = a\, x[t]$$
$$y[n] = a \cdot x[n]$$

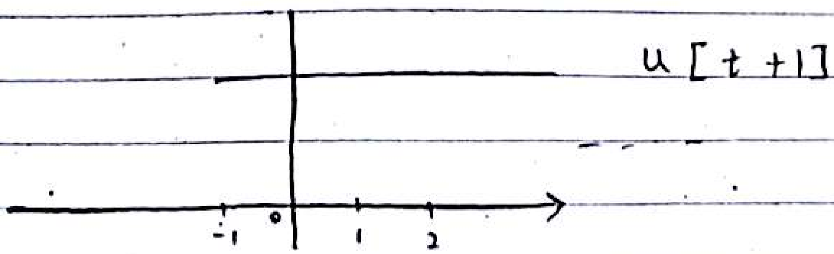
Q. Sketch the signal. $3\, x(t+2) \cdot u(1-t)$

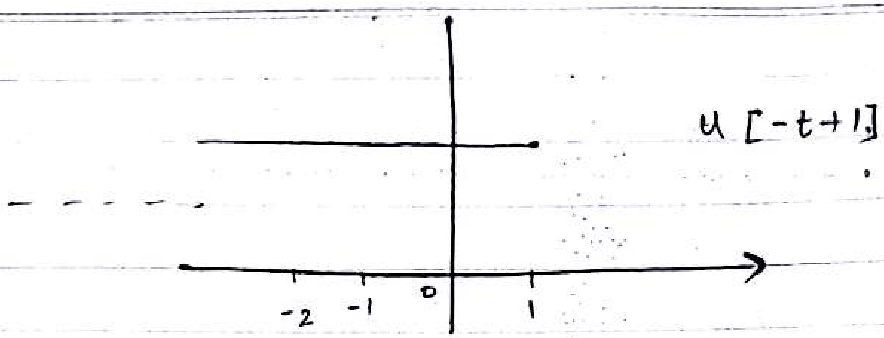


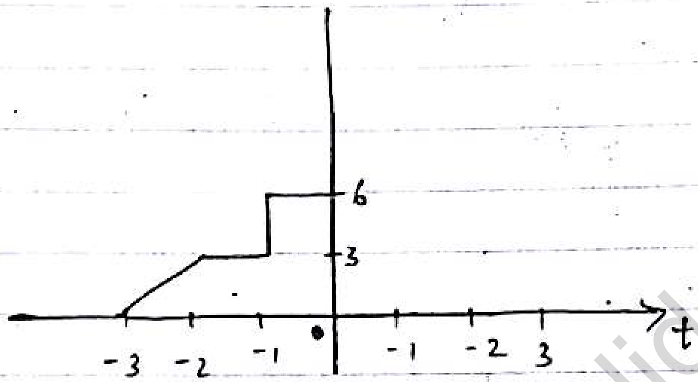








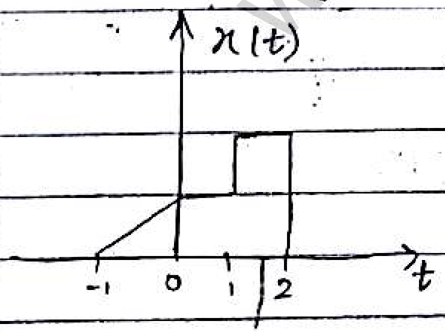


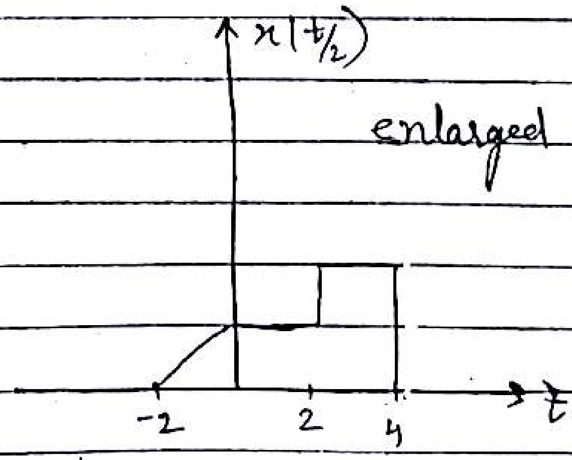


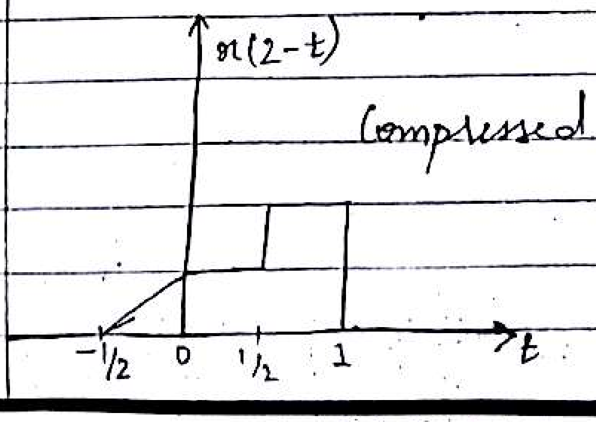
* Time Scaling

The scaling of a signal can be represented by replacing t or n by a.t / a.n in signal x t or x n i.e.,

$$y(t) = x(a.t)$$
$$y[n] = x[a.n]$$





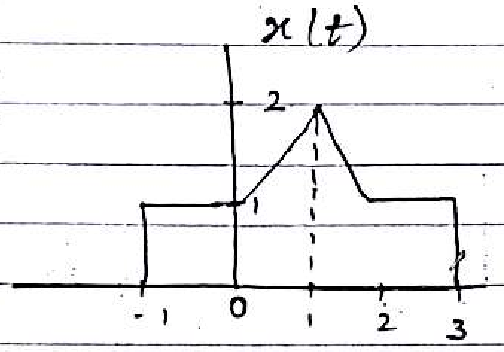


Q: For given signal $x(t)$ plot the following

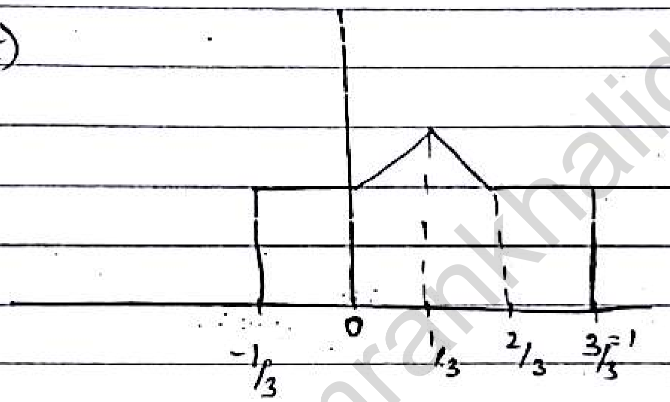
i) $x(3t)$ ii) $x\left(\frac{t}{3}\right)$ iii) $2 \cdot x(t/2)$

iv) $x\left(\frac{t-1}{2}\right)$

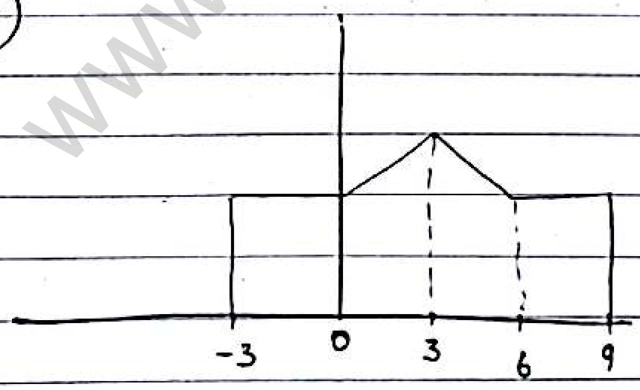
v) $x(2t-1)$



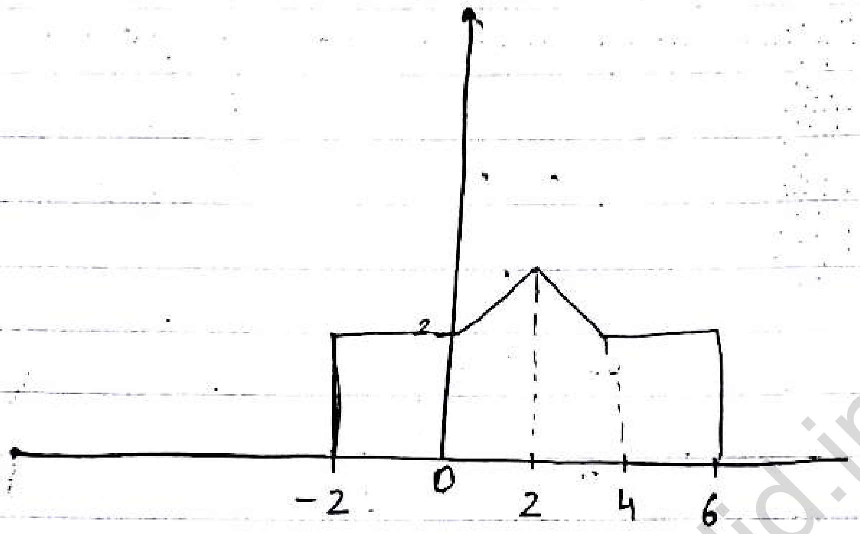
① $x(3t)$



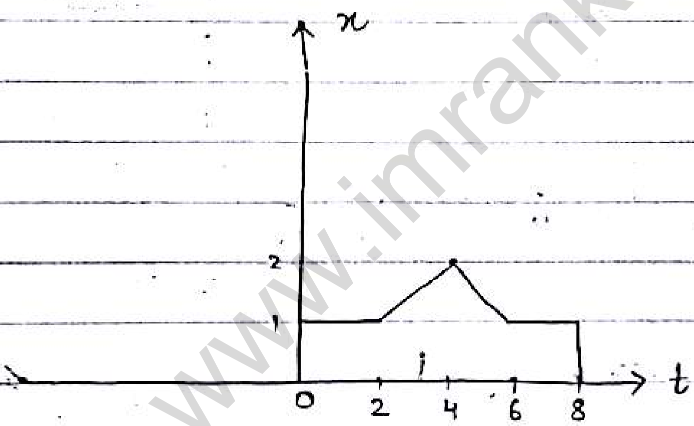
② $x\left(\frac{t}{3}\right)$



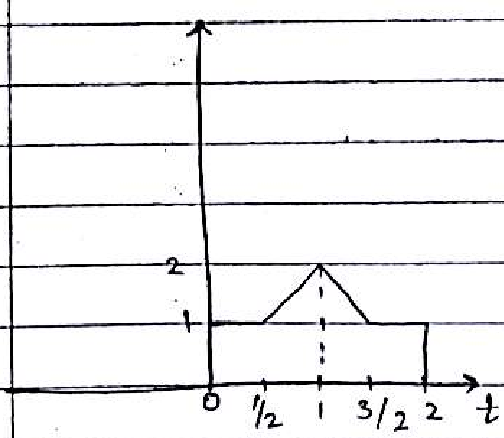
③ $2 \cdot x\left(\frac{t}{2}\right)$



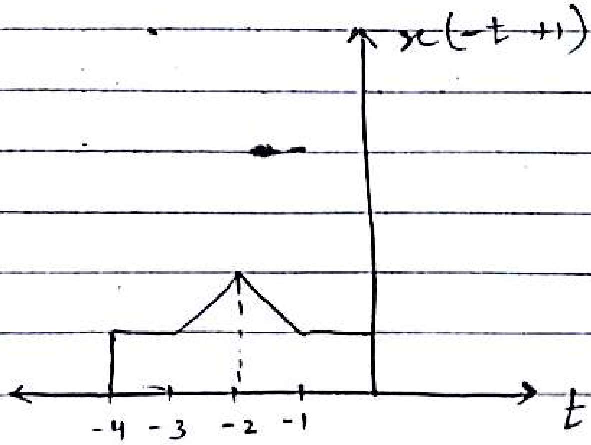
④ $x\left(\frac{t}{2} - 1\right)$



⑤ $x(2t - 1)$



⑥ $x(-t-1) = x(-(t+1))$



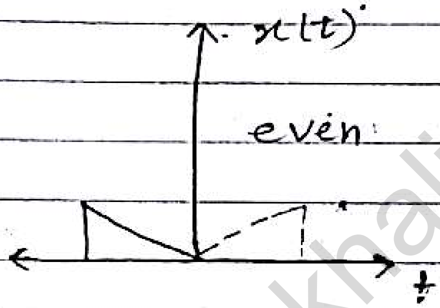
* Even & Odd signals

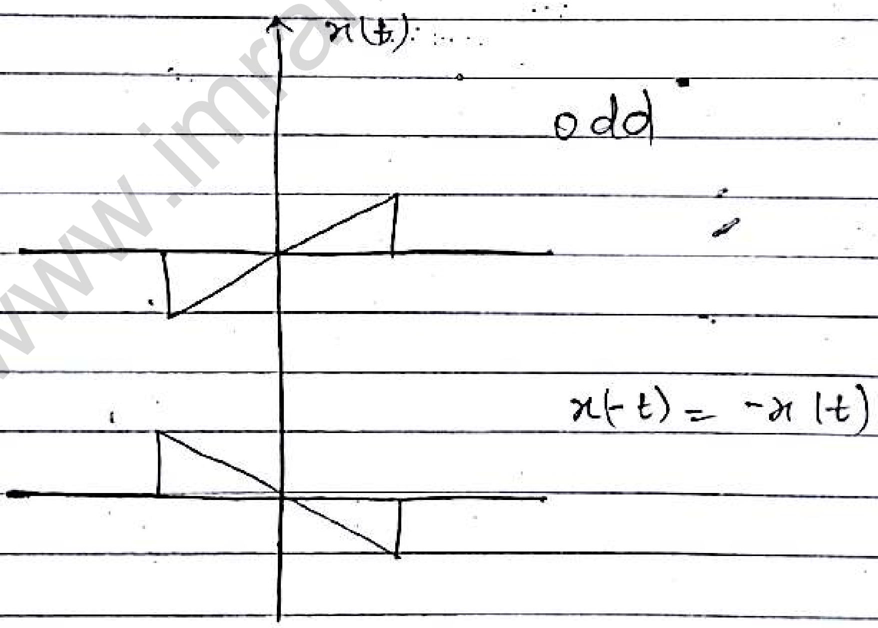
Signal $x(t)$ or $x(n)$ is ~~reffere~~ referred to an even signal if $x(-t) = x(t)$

$$x[-n] = x[n]$$

Signal $x(t)$ or $x(n)$ is referred to as an odd signal if $x(-t) = -x(t)$

$$x[-n] = -x[n]$$





Any signal $x(t)$ or $x(n)$ can be expressed as a sum of 2 signals one of which is even and one of which is odd i.e.

$$x(t) = x_e(t) + x_o(t)$$

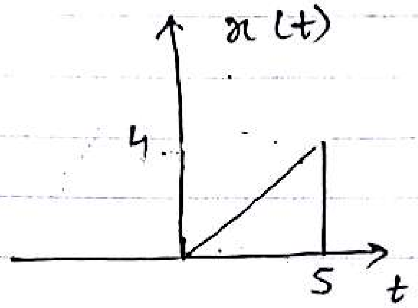
$$x[n] = x_e[n] + x_o[n]$$

where, $x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right]$

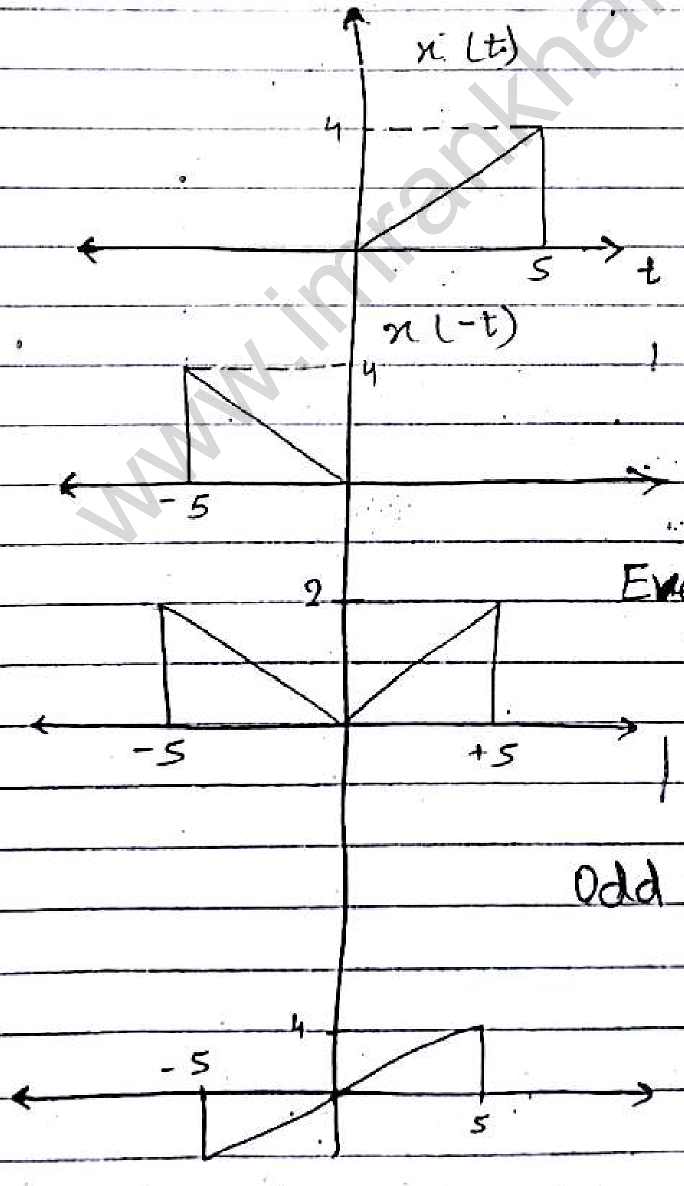
$$x_e[n] = \frac{1}{2}\left[x[n] + x[-n]\right]$$

$$x_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right]$$

Q.



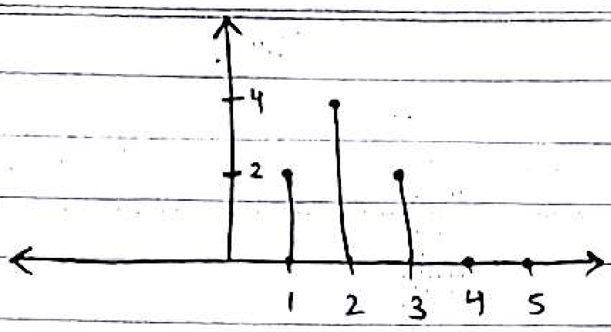
Find even & odd component.



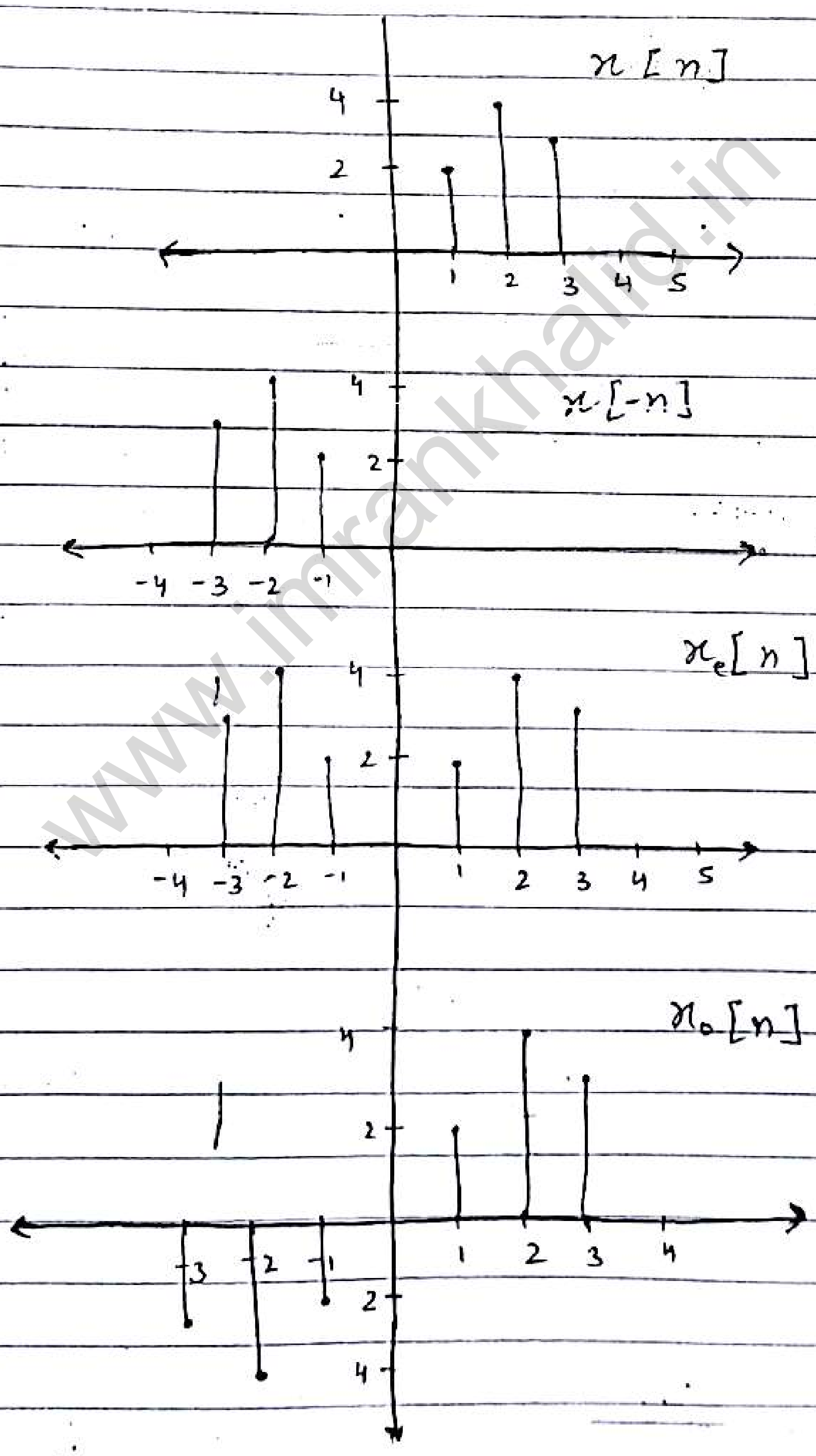
Even component

Odd component

Q.



Sketch $x_e[n]$
$x_o[n]$



Q Find the even and odd component of the following signal.

i) $x(t) = \cos t + \sin t + (\cos t \cdot \sin t)$

$$x(-t) = \cos(-t) + \sin(-t) + \cos(-t)\cdot\sin(-t)$$
$$= \cos t - \sin t - \cos t \sin t$$

$$x_e(t) = \frac{1}{2}\{x(t) + x(-t)\} = \cos t$$

$$x_o(t) = \frac{1}{2}\{x(t) - x(-t)\}$$

$$= \frac{1}{2}\{\cancel{\cos t} + \sin t + (\cos t \cdot \sin t) - \cancel{\cos t} + \sin t + \cos t \sin t$$

$$= \frac{1}{2}\{2 \sin t + 2 \sin t \cos t\}$$

$$= \sin t + \cos t \sin t$$

ii) $x(t) = \sin t + 2 \sin t + 2 \sin^2 t \cos t$

$$x(-t) = -\sin t - 2 \sin t + 2 \sin^2 t \cos t$$

$$x_e(t) = \frac{1}{2}\{x(t) + x(-t)\}$$

$$= \frac{1}{2}\{\cancel{\sin t} + 2\cancel{\sin t} + 2\sin^2 t \cos t - \cancel{\sin t} - 2\cancel{\sin t} + 2 \sin^2 \cos t\}$$

$$= \frac{1}{2}\{4 \sin^2 t \cos t\}$$

$$= 2 \sin^2 t \cos t$$

$$x_o(t) = \frac{1}{2}\{2\sin t + 4\sin t\}$$

$$= 3\sin t$$

Q. $x[n] = \{-2, 1, \underset{\uparrow}{2}, -1, 3\}$

$x[-n] = \{3, -1, \underset{\uparrow}{2}, 1, -2\}$

$$x_{e_0}[n] = \frac{1}{2}[2+2] = 2$$

$$x_{e_1}[n] = \frac{1}{2}[-1+1] = 0$$

$$x_{e_2}[n] = \frac{1}{2}[3-2] = \frac{1}{2}$$

$$x_{e(-1)} = \frac{1}{2}[1+(-1)]$$

$$= 0$$

$$x_{e(-2)} = \frac{1}{2}[-2+3]$$

$$= \frac{1}{2}$$

$$x_e(t) = \left(\frac{1}{2}, 0, 2, 0, \frac{1}{2}\right)$$

Q. $x[n] = \{1, \underset{\uparrow}{0}, -1, 2, 3\}$

$x[-n] = \{3, 2, -1, \underset{\uparrow}{0}, 1\}$

$x_e(n) = \{-3, -2, -1, 0, 1, 2, 3\}$

$\cancel{x_{e_0}(n) = \frac{1}{2}[0+2] = 1}$

$\cancel{x_{e_1}(n) = \frac{1}{2}[}$

$\cancel{x_e(n) = \{-\frac{3}{2}, -1,}$

Q. $x[n] = \{1, \underset{\uparrow}{0}, -1, 2, 3\}$, find $x_e(n)$ & $x_o(n)$

$x[-n] = \{3, 2, -1, \underset{\uparrow}{0}, 1\}$

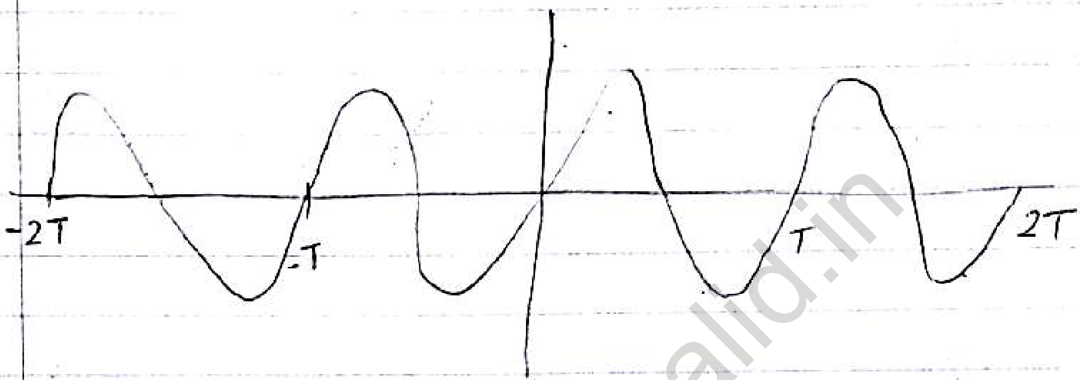
$x_e(n) = \{\frac{3}{2}, 1, 0, \underset{\uparrow}{0}, 0, 1, \frac{3}{2}\}$

$x_o[n] = \{-\frac{3}{2}, -1, 1, \underset{\uparrow}{0}, -1, 1, \frac{3}{2}\}$

22/Aug/2016

Periodic and Non-periodic Signals

A continuous time signal $x(t)$ is said to be periodic with T if there is a positive non-zero value of T for which there is $x(t+T) = x(t)$ for all t



for given fig.

$$x(t+mT) = x(t)$$
for all T & integer m

The signal which is not periodic is called aperiodic or non-periodic signal.

Condition for sum of 2 periodic signals to be periodic.

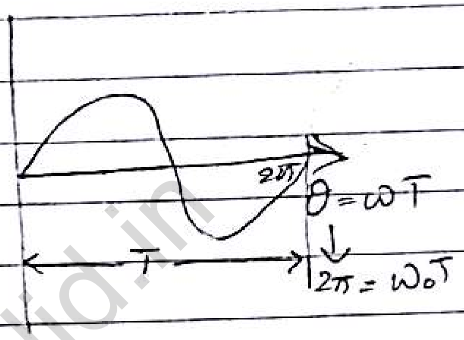
The sum of 2 periodic signals $x_1(t)$ & $x_2(t)$ with periods T_1 & T_2 may or may not be periodic depending on the relation b/w T_1 & T_2. If the sum to be periodic then the ~~sad~~ ratio of periods $\frac{T_1}{T_2}$ must be a rational no. or ratio of 2 integers. Otherwise, the sum is non periodic.

Q. Find whether the following signals are periodic or not.

$$x(t) = 2\cos(10t+1) - \sin(4t-1)$$

$$\omega_0 T = 2\pi$$

$$T_1 = \frac{2\pi}{\omega_0}$$



* $x(t)$ is periodic, if $x(t+T) = x(t)$

$$x(t) = \sin \omega t \quad \text{--- (i)}$$
$$x(t+T) = \sin[\omega(t+T)]$$
$$= \sin(\omega t + \omega T) \quad \text{--- (ii)}$$

$$eq^n \text{ (i)} = eq^n \text{ (ii)}$$
$$\text{when } \omega T = 2\pi m$$
$$T = \frac{2\pi m}{\omega}$$
$$\sin(2\pi m + \theta) = \sin\theta$$

Q. Let two signals $x_1(t)$ & $x_2(t)$ are periodic signal, find the condition for which the sum of given two periodic signals is periodic?

$\rightarrow$ $x(t) = x_1(t) + x_2(t)$ is periodic

$x_1(t)$ is periodic means $x_1(t) = x_1(t+T_1)$
$= x_1(t + mT_1)$

$x_2(t)$ is periodic means $x_2(t) = x_2(t + nT_2)$

Now, if $x(t)$ is periodic
then $x(t) = x(t+T)$

Assume, T_1 & T_2 are such that $T = mT_1 = nT_2$

$$x(t+T) = x_1(t+T) + x_2(t+T)$$
$$x(t+T) = x_1(t+mT_1) + x_2(t+nT_2)$$
$$= x_1(t) + x_2(t)$$
$$= x(t)$$

$\therefore$ Condition for $x(t)$ to be periodic is $T = mT_1 = nT_2$

$$\Rightarrow \quad \frac{T_1}{T_2} = \frac{n}{m} = \text{rational no. where } n \,\&\, m \text{ are integers.}$$

Q. Find the condition for which $x_n = e^{j\omega_0 n}$ is periodic in terms of fundamental period

Sol^n

If $x[n]$ is periodic

then, $x[n] = x[n+N]$

$$e^{j\omega_0 n} = e^{j\omega_0(n+N)}$$

$$e^{j\omega_0 n} = e^{j\omega_0 n} \cdot e^{j\omega_0 N}$$

$$\therefore \quad x[n] = x[n+N]$$

$$e^{j\omega_0 N} = 1$$

$$e^{j\omega_0 N} = e^{j2\pi m}$$

$$\omega_0 N = 2\pi m$$

$$N = \left(\frac{2\pi}{\omega_0}\right) m$$

$$\Rightarrow \quad \boxed{\frac{\omega_0}{2\pi} = \frac{m}{N}} = \text{rational no.}$$

Q. Find whether the following signals are periodic or not also find fundamental period if the signal is periodic.

i)

$$x[n] = e^{j7\pi n}$$

$$= e^{j\omega_0 n}$$

$$\omega_0 = 7\pi$$

$\sin \omega_0 n$

$\sin 7\pi n$

$$\therefore \quad \frac{\omega_0}{2\pi} = \frac{7\pi}{2\pi} = \frac{7}{2}$$

$$N = 2$$

ii) $x[n] = 3e^{j5(n+\frac{1}{2})}$

$$x[n] = e^{j\omega_0 n}$$

$$x[n] = 3e^{j5(n+\frac{1}{2})}$$

$$= 3e^{j(5n+\frac{5}{2})}$$

$$= 3e^{j5(n+\frac{1}{2})}$$

$$\Rightarrow \omega_0 = 5$$

$$\frac{\omega_0}{2\pi} = \frac{5}{2\pi}$$

Since, ratio is irrational
$\therefore$ it is non-periodic.

Q. $x[n] = \sin\left[\frac{6\pi}{7}n + 1\right]$

We know that $\sin(\omega_0 n + \theta)$

$$\omega_0 = \frac{6\pi}{7}$$

$$\frac{\omega_0}{2\pi} = \frac{6\pi/7}{2\pi} = \frac{3}{7}$$

$$N = 7$$

Q. $x[n] = e^{j(\frac{2\pi}{3})n} + e^{j(\frac{3\pi}{4})n}$

$$= x_1[n] + x_2[n]$$

$x_1[n] = e^{j(\frac{2\pi}{3})n}$	$x_2[n] = e^{j(\frac{3\pi}{4})n}$
$\omega_0 = 2\pi/3$	$\omega_0 = 3\pi/4$
$\frac{\omega_0}{2\pi} = \frac{2\pi/3}{2\pi} = \frac{1}{3}$	$\frac{\omega_0}{2\pi} = \frac{3\pi/4}{2\pi} = \frac{3}{8}$
$N_1 = 3$	$N_2 = 8$
$\frac{N_1}{N_2} = \frac{3}{8}$	